Sustainable Vehicle-Routing Problem with Time Windows by Heterogeneous Fleet of Vehicles and Separated Compartments: Application in Waste Collection Problem

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Abstract
This study aims to solve a sustainable vehicle routing problem (VRP), where special characteristics such as mixed close and open routes, and several servicing depots in different areas are considered for achieving to real life conditions. Fleets of vehicle are heterogeneous, and specified capacity for each type of waste is assumed in this paper. Vehicles have different attributes such as limitations on traveling time, fixed and variable cost, and amount of pollutants emitted to an environment. For achieving a sustainable collection network, economic, environmental, and social aspects should be considered, simultaneously; therefore, in this paper three different objective functions are optimized at the same time. The first one minimizes the cost of collecting wastes from customers’ location. The second one minimizes the pollutants emitted from vehicles while they are collecting wastes, and finally the third one minimizes violation from servicing time limitations. A new mathematical mixed integer programming model is developed for solving this problem, and the problem is tackled by CPLEX solver and augmented $\varepsilon$-constraint method. Moreover, AHP technique is applied in order to help us to choose the best decision. Finally, sensitivity analysis is done on some important parameters.

Keywords: Waste collection, close-open mixed VRP, augmented $\varepsilon$-constraint, AHP technique, sensitivity analysis.

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1. Introduction

Management of delivering and distributing goods has become an interesting and important issue in the past decades, and it has attracted a researchers’ attention. Generally, in strategic and tactical levels of distribution organizations, finding routes that impose less cost to organization is an important and critical decision [Hiermann et al. 2016]. However, for achieving a sustainable supply chain, paying attention to only economic aspects are not sufficient. In a sustainable supply chain networks, economic, environmental, and social aspects should be considered, simultaneously. In economic aspect, recent studies have showed that using optimization procedures for planning a distribution process causes to saving transportation costs in the range of 5 to 20 percent [Penna et al. 2016]. The vehicle routing problem (VRP) is one of the most challenging optimization problems. A VRP consists of finding a set of routes based on the location of customers and depots for a number of vehicles visiting customers and depots, while minimizing the overall cost [Errico et al. 2016]. The classical formulation of VRP was introduced by Dantzig and Ramser (1959) for the first time, and then Flood (1956) assumed it as a generalized version of the Traveling Salesman Problem (TSP).

Recently, an increasing attention can be found to environmental aspects in designing process of logistics networks. The fact that current distribution and production strategies are not sustainable motivates researchers to incorporate green concepts in their studies. Supply chain networks should pay special attention to environmental aspects. It is needed to change transportation plans, and shift them to a sustainable network designs with fewer negative impact on environment. Wide variety of approaches concerning green aspects exist in the literature. For example, using intelligent transportation systems, electric vehicles, alternative fuels such as biomasses, and other eco-friendly infrastructures. Some of recent studies considered different strategies in order to reach to sustainability are Leggieri and Haouari (2017), Çimen and Soysal (2017) and Baraki and Kianfar (2017).

Finally for social aspect, in this study, time windows constraint in customer’s location is considered in which arriving sooner or later than specified time are not acceptable and will pose penalty to system. Number of studies can be found in this area such as [Pecin et al. 2017 and Alinezhad et al. 2018]. All in all, the purpose of this study is achieving a trade-off between mentioned three conflicting objectives.

2. Literature Review

As an aforementioned, three aspects of supply chain will be discussed in this study, so literature review is divided into three sections. At first stage, we will discuss vehicle routing problem aspects.

There are different categories for VRP, and scholars paid attention to them. One of the most important variants of this aspects can be found in Marinakis and Marinaki (2010) study. VRP with Time Windows (VRPTW) is one variant of this problem in which vehicles should service to customers during a specific time range [Azi, Gendreau, and Potvin, 2010]. Heterogeneous fleet VRP (HVRP) in another type, where the vehicles servicing to the customers have different capacities [Azadeh and Farrokh-Asl. 2017; Rabbani, Farrokhhi-asl, and Rafiei, 2016]. VRP problems based on number of depots is divided into two groups including single depot VRP and multi depot VRPs. Many studies have paid attention to single depot VRP, whereas multi
depot VRPs are much closer to real world problems, because companies have often more than one depot [Wang et al. 2016]. In multi depot vehicle routing problem (MDVRP), the critical decision is assigning customers to more than one depot. In addition to assigning customers to depots, optimal routes should be found in order to delivering customer’s demands [Azadeh and Farrokhi-Asl, 2017]. Kim, Sun, and Lee (2013) have focussed on single depot problem. Rabbani, M., Farrokhi-asl, H., and Rafiei, H. (2016), Cordeau and Maischberger (2012), Escobar, Linfati, Baldoquin (2014), Shimizu and Sakaguchi (2014), Subramanian, Uchoa, and Ochi (2013), and Vidal et al. (2014) are researchers that had addressed multiple depots in their studies so far. For solving this type of problem, many heuristic methods have been used by mentioned researchers. For instance, Rabbani, M., Farrokhi-asl, H., and Rafiei, H. (2016) used a hybrid genetic algorithm. Cordeau and Maischberger (2012) and Escobar, Linfati, Baldoquin (2014) implemented tabu search (TS) algorithm for solving their problem; Shimizu and Sakaguchi (2014) solved MDVRP problem with the hierarchical hybrid meta-heuristic; Subramanian, Uchoa, and Ochi (2013) used local search for solving the problem; Vidal et al. (2014) used genetic algorithm. Regarding different types of vehicle’s possession, VRP is divided into three categories including close, open and mixed close open. In close VRP all of the vehicles belong to a company and after servicing to the customers should come back to the same depot. In open VRP, the vehicles do not belong to the company (e.g. the vehicles are hired) and after servicing to the last customer in their routes they are free [Yakici, 2017]. Li, Golden, and Wasil (2007) used record-to-record travel algorithm for solving an OVRP problem. Repoussis et al. (2010) developed a hybrid evolution strategy for OVRP. Most complicated type in this class of problems is close open mixed VRP (COMVRP). In COMVRP some of the vehicles belong to the company and the rest are not. Close-open mixed VRP is closer to the real life problems, because fluctuation in demands caused to fluctuation in numbers of vehicles which are needed for answering to all of the demands. As a result, these companies need to have a large number of vehicles to respond maximum demand or have less vehicles, but respond to exceeded demand by hiring some vehicles which is called COMVRP [Azadeh and Farrokhi-Asl, 2017]. For instance, Liu and Jiang (2012) mentioned a real case of COMVRP which was used in a chemical industry in Shanghai, China. In this study, heterogeneous fleet of vehicle is considered where the vehicles have limitation on maximum capacity and operating time. In recent researches, some of the researchers such as Rabbani, Farrokhi-asl, and Rafiei, (2016), Dominguez et al. (2016) and Mancini (2016) paid attention to this aspect of VRP. As such, we try to address different aspects of VRPs, but the applications of these types of problems are various. For example Issabakhsh et al. (2018) considered different applications of VRP in health care, or study of Ghannadpour and Zarrabi (2017) in railroad scheduling.

There are different types of problems defined by combination of VRP and some special consideration(s) a number of which will be discussed in the following sections. If in VRP location of facilities be question of problem, we are encountering a new type of problem which is called location routing problem (LRP). Ghatreh Samani and Hosseini-Motlagh (2017) study is an instance of this type of problem. If in VRP; inventory of goods be question of problem besides routing of vehicles, the new it is called inventory routing problem (IRP). The study of Nikkhah Qamsari, Hosseini Motlagh, and Jokar (2017) can be mentioned in discussed area.
In recent decades, the importance of CO2 emissions in VRP has attracted much attention. The reason is that the people and governments are persuading companies to reduce CO2 emission. People by supporting green companies and government by passing many laws support green companies and motivate them to moving toward greenness [Koç and Karaoglan, 2016]. Some of studies about green vehicle routing problem (GVRP) are as follows: Felipe et al. (2014) used several heuristic methods for solving a problem in which a fleet of vehicles was electric vehicles. Erdoğan and Miller-Hooks (2012) implemented a mixed integer programming for solving GVRP. Jemai, Zekri, and Mellouli (2012) aimed to find an optimal route with minimum traveled distance and minimum CO2 emission. They implemented NSGA-II method.

Finally for social consideration, we use time window on customer’s locations. Satisfying customer’s demands with a constraint on servicing time relates to a new type of VRPs called VRP with time windows (VRPTW). In this case, for determining the optimal fleet routes of vehicles the decision maker should consider limitation on time that servicing should start in this limitation [Hernandez et al.2016]. In transportation, logistics, and distribution problems for reaching to more realistic situations, considering limitation on servicing time is advised [Bae and Moon, 2016]. Some recent studies considered time window are Koç et al. (2016), and Braaten et al. (2017).

Generally, waste collection problem can be classified into three categories including industrial, commercial and residential waste collection problems. Industrial waste collection problem called rollon-rolloff problem is composed of pickup, transportation, unloading and dumping of big containers which may be found in shopping centers or construction sites [Farrokhi-Asl, Makui, Ghoussi and Rabbani, 2018]. In commercial waste collection, wastes are collected from commercial locations and each commercial location is considered as a transportation network node in node based routing problem. Finally, in residential waste collection, wastes should be collected from household garbage along the streets. Residential waste collection is considered as an arc routing problem. Wy and Kim (2013) proposed a hybrid meta-heuristic approach consisting of various improvement methods and a large neighborhood search for solving rollon-rolloff problem. In managing hazardous wastes, collecting, transporting and disposing of garbage which are hazardous for people and environmental health is considered. Alumur and Kara (2007) proposed a new model for hazardous waste collection routing problem. They discussed in their paper about specifying where disposal facilities should be opened, which level of technology should be established, and how to route the different types of hazardous wastes.

The problem presented in the current study is a combination of multi depot vehicle routing problem and close-open mixed vehicle routing problem. Vehicles are heterogeneous; that is, each type of vehicle has different maximum traveling time, fixed and variable cost and capacity for each type of waste. In this study, some vehicles should return to relative depot after servicing to customers where they had departed from and rest of them will not come back. Each type of waste should be disposed in a compatible disposal facility. It is assumed that customers generate all types of wastes, so each vehicle should move to all disposal facilities. These features help the problem to get closer to real life problems. One of these features is considering both internal and external fleet of vehicles. In most companies they have their own fleet of vehicles but sometimes the number of own
vehicles of company which are called internal fleet, cannot serve all of the customer demands, so external fleet will be needed to preform rest of the customer demands. Another feature is considering multi-depot problem. However most of papers paid attention to single-depot problem, but in real cases often we confront with companies which have more than one depot. Each type of vehicle based on its technology level has specified fuel consumption per unit of time, and this energy consumption has direct relationship with amount of pollutant which the vehicle will emit. A case study is considered and for solving this case GAMS software with CPLEX solver had been used. At the end of the paper sensitivity analysis on some parameters had been done.

Table 1. A Literature Overview in the COMVRP and MDVRP.

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<th>Deterministic Demand</th>
<th>Open VRP</th>
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<th>Close open mixed VRP</th>
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Briefly, we can say that solving a sustainable vehicle routing problem with three conflicting objectives including economic, environmental and social, and considering many different assumption such as multi depot VRP, Close open mixed VRP, heterogeneous fleet of vehicle with separated compartment, customer’s different importance, using \(\varepsilon\)-constraint method for achieving non-dominated solutions, using AHP technique for choosing best solution between numerous non-dominated solutions and doing sensitivity analysis for solving the problem which is a relatively rare in literature; are contributions of authors in this study.

The rest of sections in this paper are as follows: In section 2, mathematical model is developed. In section 3, proposed methodology is described. In section 4, the numerical results is reported, and finally in section 5 conclusion and future remarks are reviewed.

3. Problem Description

Waste collection problem is one of the most attractive operational research (OR) problems, and is very critical for human life and health. In this problem, wastes are left at specific sites by customers, and they request for collecting them. It is assumed that wastes are separated by customer, because of facilitating of recycling process. In this sites, wastes are separated to different types such as glasses, foods, papers, bottles, plastics and etc. For serving customer demands internal and external fleet of vehicles are used (i.e., a company decides to get help from contractors). Company has limited internal vehicles and the number of external vehicles is assumed to be unlimited. Both internal and external fleet have variable cost and just external fleet has fixed cost. Vehicles of internal fleet after serving customers should come back to the depot, but vehicles of external fleet are free after finishing a service to last customer. Each vehicle has multiple compartments corresponding to one type of wastes and the capacity of these compartments are limited. Vehicles of internal fleet start their trip at depot and visit customers and collect their wastes regard to their capacity or their time limitation and after collecting wastes from last customer they will visit all of the disposal facility. Finally, after last disposal facility, they will come back to the depot. The differences between internal and external fleet are: (1) externals start their trip at first customer location (no at depots) and (2) after last disposal facility they do not go to depots.

The objectives of this problem are minimizing traveling costs, reducing CO2 emission, and trespassing from allowed service time. A MDVRP with COMVRP and heterogeneous fleet of vehicles are considered in this paper, so generally we can called it as COMDVRP. In Figure 1, two type of trips are shown in which route 1 is close and route 2 is open.

3.1 Mathematical Formulation

Assumptions:

- Each customer has request for collecting all types of wastes.
- Each type of wastes should left at compatible disposal facility.
- Traveling between nodes and customer’s demand are deterministic.
- Number of external vehicles are unlimited and internals vice versa.
- Internal and external fleet of vehicles are heterogeneous.
- For each type of waste each vehicle has separated compartment.
- Both internal and external fleets have variable cost and only internals have fixed cost.
Indices:
D = (1,2,..., d) Set of indexes for depots
K = (1,2,..., k) Set of indexes for vehicles
F = (1,2,..., f) Set of indexes for disposal facilities
C = (1,2,..., c) Set of indexes for customers
W = (1,2,..., w) Set of indexes for type of wastes
S = (1,2,..., s) Set of indexes for internal/external fleet

Parameters:
d_{iw} : demand of customer i for collection of waste type w
C_{wk} : Capacity of vehicle k for waste type w
e_i : Earliest time at customer i
l_i: Latest time at node i
p_e: Penalty for arrive early at customer location
p_l: Penalty for arrive late at customer location

AT_k : Maximum allowable traveling time for vehicle k
I : Maximum internal vehicle
t_{ij} : traveling time between node i and node j
LU_{iwsk} : loading/unloading time for waste type w by means of vehicle k of fleet s in node i
f_k : fixed cost of using vehicle k
V_k : Variable cost of using vehicle k
a_k : Amount of pollutants which are emitted by vehicle k per unit of time
M: great number

Decision variables:
x_{ijk} : If vehicle k belonged to fleet type s travels directly from node i to node j, x_{ijk}= 1; otherwise = 0
Q_{isk} : If vehicle k of fleet type s departs from depot, Q_{isk} = 1; otherwise = 0
y_{isk} : If vehicle k of fleet s is allocated to customer i , y_{isk} = 1; otherwise = 0
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\( A_{iskw} \) : Continuous variable that represents the load of compartment \( w \) of vehicle \( k \) and type \( s \) just after leaving the customer \( i \)

\( G_{isk} \) : Continuous variable that represents the traveling time of vehicle \( k \) of fleet \( s \) from depot to the customer \( i \) when it arrives at customer \( i \)'s location

Objective functions:

\[
\begin{align*}
\text{Min} & \sum_{k \in K} f_k \sum_{i \in D} Q_{ik} + \sum_{k \in K} V_k \sum_{s \in S} \sum_{i \in D} \sum_{j \in D \cup F} \sum_{w \in W} t_{ij} x_{ijsk} - \sum_{k \in K} V_k \sum_{i \in F} \sum_{j \in D} t_{ij} x_{ij1k} \\
& + \sum_{k \in K} V_k \sum_{s \in S} \sum_{i \in D \cup F} \sum_{j \in D \cup F} \sum_{w \in W} LU_{iwsk} x_{ijsk} \\
& (1)
\end{align*}
\]

\[
\begin{align*}
\text{Min} & \sum_{k \in K} a_k \sum_{s \in S} \sum_{i \in D \cup F} \sum_{j \in D \cup F} t_{ij} x_{ijsk} \\
& (2)
\end{align*}
\]

\[
\begin{align*}
\text{Min} & \sum_{i \in C} cw_i g_i \\
& (3)
\end{align*}
\]

Constraints:

\[
\begin{align*}
\sum_{s \in S} \sum_{k \in K} \sum_{i \in D \cup F} x_{ijsk} & = 1 \quad \forall j \in C \\
& (4)
\end{align*}
\]

\[
\begin{align*}
\sum_{s \in S} \sum_{k \in K} \sum_{i \in D \cup F} x_{ijsk} & = 1 \quad \forall j \in C \\
& (5)
\end{align*}
\]

\[
\begin{align*}
\sum_{i \in D \cup F} x_{ijsk} & = \sum_{i \in D \cup F} x_{ijsk} \quad \forall j \in C, s \in S, k \in K \\
& (6)
\end{align*}
\]

\[
\begin{align*}
\sum_{i \in D \cup F} x_{ijsk} & = \sum_{i \in D \cup F} x_{ijsk} \quad \forall j \in F, s \in S, k \in K \\
& (7)
\end{align*}
\]

\[
\begin{align*}
\sum_{i \in D} \sum_{j \in D \cup F} x_{ij0k} & = 0 \quad \forall s \in S, k \in K \\
& (8)
\end{align*}
\]

\[
\begin{align*}
G_{isk} & = 0 \quad \forall i \in D, s \in S, k \in K \\
& (10)
\end{align*}
\]

\[
\begin{align*}
G_{isk} + t_{ij} + \sum_{w \in W} LU_{jwsk} x_{ijsk} - M(1 - x_{ijsk}) & \leq G_{jsk} \quad \forall i, j \in C \cup F, s \in S, k \in K \\
& (11)
\end{align*}
\]

\[
\begin{align*}
G_{i0k} + t_{ij} - M(1 - x_{ij0k}) & \leq AT_k \quad \forall j \in D, i \in F, k \in K \\
& (12)
\end{align*}
\]

\[
\begin{align*}
G_{i1k} + t_{ij} + \sum_{w \in W} LU_{jw1k} x_{ij1k} - M(1 - x_{ij1k}) & \leq AT_k \quad \forall i, j \in C \cup F, k \in K \\
& (13)
\end{align*}
\]

\[
\begin{align*}
0 & \leq G_{isk} \leq \sum_{j \in D \cup F} x_{ijsk} AT_k \quad \forall i \in C, s \in S, k \in K \\
& (14)
\end{align*}
\]

\[
\begin{align*}
0 & \leq G_{isk} \leq \sum_{j \in D \cup F} x_{ijsk} AT_k \quad \forall i \in F, s \in S, k \in K \\
& (15)
\end{align*}
\]

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\[
\sum_{k \in K \cap j \in C} x_{ijok} \leq I \tag{16}
\]
\[
\sum_{j \in C \cup D} x_{isk} \leq y_{isk} \quad \forall s \in S, k \in K \tag{17}
\]
\[
\sum_{ \in C \cup D} d_{iw} y_{isk} \leq c_{wk} \quad \forall w \in W, s \in S, k \in K \tag{18}
\]
\[
A_{iskw} = 0 \quad \forall i \in D, s \in S, k \in K, w \in W \tag{19}
\]
\[
A_{iskw} + d_{jw} - M \left(1 - x_{isk} \right) \leq A_{jskw} \quad \forall i, j \in C, s \in S, k \in K, w \in W \tag{20}
\]
\[
d_{iw} \leq \sum_{s \in S} \sum_{k \in K \cap j \in C \cup D} A_{iskw} \leq \sum_{s \in S} \sum_{k \in K \cap j \in C \cup D} x_{isk} c_{wk} \quad \forall i \in C, w \in W \tag{21}
\]
\[
\sum_{s \in S} \sum_{k \in K \cap j \in C \cup D} x_{isk} = 0 \tag{22}
\]
\[
\sum_{s \in S} \sum_{k \in K \cap j \in C \cup D} x_{isk} = 0 \tag{23}
\]
\[
\sum_{s \in S} \sum_{k \in K \cap j \in C \cup D} x_{isk} = \sum_{s \in S} \sum_{k \in K \cap j \in C \cup D} x_{isk} \quad \forall h \in F \tag{24}
\]
\[
\sum_{i \in C \cup D \cup F} x_{isk} = 1 \quad \forall j \in F, s \in S, k \in K \tag{25}
\]
\[
\sum_{j \in C \cup D} x_{isk} = 1 \quad \forall i \in F, s \in S, k \in K \tag{26}
\]
\[
\sum_{j \in C \cup D} x_{isk} = Q_{isk} \quad \forall i \in D, s \in S, k \in K \tag{27}
\]
\[
c_{wi} = \frac{\sum_{w \in W} d_{iw}}{\sum_{i \in C} \sum_{w \in W} d_{iw}} \tag{28}
\]
\[
g_i = \left[\max(0, e_i - g_{isk} - L_{iskw} p_e)\right] + \left[\max(0, G_{isk} + L_{iskw} - l_i) p_i\right] \tag{29}
\]
\[
x_{isk} = \{0,1\} \quad \forall i, j \in D \cup C \cup F, s \in S, k \in K \tag{30}
\]
\[
Q_{isk} = \{0,1\} \quad \forall i \in D, s \in S, k \in K \tag{31}
\]
\[
y_{isk} = \{0,1\} \quad \forall i \in C, s \in S, k \in K \tag{32}
\]
\[
A_{iskw} \geq 0 \quad \forall i \in D \cup C, s \in S, k \in K, w \in W \tag{33}
\]
\[
G_{isk} \geq 0 \quad \forall i \in D \cup C \cup F, s \in S, k \in K \tag{34}
\]
With respect to the customer’s location, fixed and variable cost of using different vehicles, objective function (1) minimizes cost of transportation between nodes. In the first term of objective function (1) external vehicles are considered in aspect of fixed cost and in this term internal vehicles are ignored because they do not have fixed cost. In the second term variables cost for both internal and external vehicles are calculated. In the third term cost of returning external vehicles to depots is lessoned because after visiting last disposal facility they do not return to depot. Finally the last but not least term cost of the time consumed by vehicles in loading/unloading procedure is added to objective function. The goal of objective function (2) is minimizing amount of pollutants which are emitted by each vehicle when they are moving between nodes. Objective function (3) wants to minimize the cost of arriving soon or late at customers location. Constraints (4) and (5) prohibit assigning each customer to more than one single route. Equation (6) represents that each vehicle when enter a node after finishing loading/unloading wastes should leave this node, and in equation (7), the same constraint for disposal facilities are developed. Equation (8) satisfies this restriction that each vehicle which is belonged to internal fleet of vehicle (s=0), should come back to a depot after servicing. Moving vehicles between depots is prohibited, this restriction is justified by constraint (9). Traveling time for arriving to each depot for starting servicing is equal to zero, and this is shown by equation (10). If vehicle type $k$ travels directly from node $i$ to node $j$, time of arriving to node $j$ is greater than time of arriving the vehicle to node $i$ plus loading/unloading time at node $i$ plus traveling time between nodes $i$ and $j$, this is shown by constraint (11). Constraint (12) is as same as constraint (11) in case of internal fleet of vehicle which should back to depot which they were departed from it after visiting last disposal facility. Constraint (13) is about internal fleet of vehicles (s=1) and set this restriction that if vehicle type $k$ travel directly from node $i$ (customer or disposal facility node) to node $j$ (as same as node $i$), time of arriving mentioned vehicle to node $i$ plus loading/unloading time at node $i$ plus traveling time between nodes $i$ and $j$ should not be greater than maximum traveling time of vehicle type $k$. If vehicle type $k$ from fleet $s$ travel directly from node $i$ (depot or customer node) to node $j$ (customer node), arrival time on node $i$ should be less than maximum traveling time of vehicle type $k$, this limitation is showed by constraint (14). Constraint (15) is as same as constraint (14) and difference is that node $i$ is used for disposal facilities and node $j$ is used for disposal facility and customer nodes. Because of time limitations which exist on each vehicle some restrictions are needed which this restriction are justified by equations (10)–(15). Equation (16) satisfy this limitation that internal vehicles have limited number. Equation (17) by specifying relationship between two variables defines a new decision variable. Equation (18) justify this restriction that collected wastes in each route should not exceed from determined capacity for each vehicle. Equations (19)–(21) are sub tour elimination constraints for classical vehicle routing problem which is proposed by Desrochers and Laporte (1991) for the first time and then revised by Kara, Laporte, and Bektas (2004). Equation (22) is restricting moving directly from customers to depots. Equation (23) is forbidding moving directly from depots to disposal facilities, because vehicles before going to depots should empty their loads at disposal facilities. Equation (24) guarantees that all the vehicles which exit from depots should visit all disposal facilities. Equation (25) justifies this restriction that each vehicle can enter each disposal facility only once. Equation (26) represent that each vehicle after
visiting each disposal facility should leave it. Equation (27) by determining relationship between two variables defines a new decision variable. Equation (28) determine a new parameter as customer weight. Equation (29) measures the penalty for violation of time window at each customer location. Constraints (30) -(34) specify the ranges of variables.

4. Methodology

In this section, the methodology which is applied for solving this model will be discussed.

4.1 The Augmented $\varepsilon$-Constraint Approach

Three objectives have to be optimized simultaneously in the proposed model in this paper. In multi-objective mathematical models, instead of finding one optimal solution, we search for a set of solutions which these solutions do not dominate each other. These non-dominated solutions for conflicting objectives are known as Pareto optimal solutions. In addition, there are three main reasons which motivate researchers to use $\varepsilon$-Constraint method [Y. Y. Haimes, Ladson, and Wismer, 1971]:

- This method can produce efficient unsupported solutions in an integer program model;
- It can include different scales for objective functions;
- By controlling grid points for each objective function this method can produce solutions which are adjusted in varieties and number. For finding non-dominated solution in multi-objective problems, the $\varepsilon$-Constraint is acknowledged method. This method proposed by Haimes, Y. Y., Ladson, L., and Wismer, D. A. (1971) for the first time, and then developed by Mavrotas (2009). In this method one of the objective functions is optimized while other objectives considered as constraints, and these constraints have upper bound ($\varepsilon_m$). In equation (35) this approach is shown.

\[
\min \left( g_1(x), g_2(x), ..., g_q(x) \right) \\
\text{s.t.} \quad x \in S
\]

where $g_1(x), g_2(x), ..., g_q(x)$ are the $q$ objective functions of problem, $x$ is the decision variables vector, and $S$ is feasible region. As mentioned in previous, in $\varepsilon$-Constraint method for solving multi-objective problems one of the objective functions will be optimized and the other objective functions are considered as constraints, as shown in the following equation:

\[
\min g_1(x) \\
\text{s.t.} \quad g_2(x) \leq \varepsilon_2 \quad g_3(x) \leq \varepsilon_3 \quad \cdots \quad g_q(x) \leq \varepsilon_q \\
x \in S
\]  

\[\text{(36)}\]

Epsilon in this method are grid point of conflict objectives which in previous step are considered as constraints. By calculating range of objective functions, we can generate payoff tables. It is plausible that weak solutions are produced by this method. Weak solution is a solution that one or more than one solution can be found that can dominate this solution, but the method considers this solution as one of the acceptable Pareto solutions. For overcoming to this deficiency augmented $\varepsilon$-Constraint has been developed by [Mavrotas and Florios, 2013]. As shown in Equation (37), this method ensures efficiency of Pareto optimal solutions.
Min\{ g_1(x) - \theta \left( \frac{P_2}{H_2} + \frac{P_3}{H_3} + \ldots + \frac{P_Q}{H_Q} \right) \}

\text{s.t.}
\begin{align*}
g_2(x) - P_2 &= \varepsilon_2 \\
g_3(x) - P_3 &= \varepsilon_3 \\
\ldots \ldots \\
g_q(x) - P_Q &= \varepsilon_q
\end{align*}

x \in S

H_2, H_3, \ldots, H_Q are range of objective functions, respectively. P_2, P_3, \ldots, P_Q are the surplus variables of the constraints, respectively and \( \theta \in (10^{-6}, 10^{-3}) \).

### 4.2 AHP Technique

As an aforementioned, by using \( \varepsilon \)-Constraint method number of solutions will be obtained which we should select one of them. Analytic hierarchy process (AHP) is one of the best techniques which helps decision makers to make decision when numerous choices are available. In this technique, with regard to alternatives and criteria, analyzing complex decisions occur. Saaty (1990) proposed this technique for the first time. In this paper, by solving the problem with \( \varepsilon \)-Constraint method, number of solutions will be obtained. Among all of the solution decision maker should select one of them. It is obvious that decision maker wants to choose the best choice which this choice has minimum travelling cost, minimum \( CO_2 \) emission, and minimum dissatisfaction of customers; however, there is no solution which have all of these criteria in the best situation, simultaneously. Applying this technique, we want to rank the solutions regard to priorities and select the solution which is gained the highest rank. For this purpose, we apply AHP technique. According to this problem we have some alternatives (different obtained solutions) and three criteria (objective functions). The AHP technique is used to sort solutions regard to their obtained score in AHP technique and finally select the solution which has the highest score in AHP.

---

**Figure 2. Alternatives and criteria in AHP technique**
Decision makers for each pair of criteria should specify which one of them is more important. By assigning a weight by decision maker, relative priority of criteria will be rated. Then, for each criterion a table like Table 2 should be made. The weights are proportional to their amount in problem; in next step we should normalized the weights and then, calculate averaged of them in order to obtain an average weight for each criteria. For each criteria a table like Table 2 should be made. Each pair of solutions should be compared in order to fill cell of the table. Cell \( (m,n) \) of table is determined by \( z_{mn}^l \) where \( l \) belongs to set of criteria and \( m \) and \( n \) belong to set of solutions.

\[
z_{mn}^l = \begin{cases} 
  v_m^l / v_n^l; & \text{if } l \text{ be positive criterion} \\
  v_n^l / v_m^l; & \text{if } l \text{ be negative criterion}
\end{cases}
\]

Amount of parameter related to solution \( m \) and criterion \( l \) is shown by \( v_m^l \). For normalization of table all cells of each column will be divide to sum of all cells of related column. In next step for calculating score of each criterion all number of each row should be summed. For calculating score of other criteria the same steps should be done. After completing this actions comparing pairs of criterion by composing a table like Table 2 should be done. Thus by doing this steps each criterion’s weight will be calculated and shown by \( z_l \). Score of each alternative is calculated by the following equation: \( Q_m = \sum_l z_m^l \times z_l \)

Score of each solution will be calculated by this manner and they will sort in decreasing order of scores. Finally the best will be selected among all of the solutions.

5. Numerical Result

Some distinct numerical problems are solved for validating the proposed model and analyzing the solutions. In this section, a case study is investigated which more descriptions about it are in the following sections.

5.1 Case Study

For solving the proposed model in this section, a case study is considered. In this case study we are encountered with a problem in which 22 municipal districts of Tehran are addressed as 22 customers and demand of each these 22 customer is known from historical data. Four disposal facilities and four depots exist, and their location are shown on Figure 3. By this map distances between customers, disposal facilities and depots can be obtained easily. In this case, four kinds of vehicles with different levels of technology are considered. Amount of CO2 emitted by each vehicle and transportation cost between nodes are gained from technical information about

<table>
<thead>
<tr>
<th>Table 2. Comparison table of AHP technique</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Criterion A</strong></td>
</tr>
<tr>
<td>Solution A</td>
</tr>
<tr>
<td>Solution B</td>
</tr>
<tr>
<td>Solution C</td>
</tr>
<tr>
<td>Solution D</td>
</tr>
<tr>
<td>Total</td>
</tr>
</tbody>
</table>
available fleets of vehicles. Additionally, two types of wastes are considered in this paper. Unloading/loading at disposal facilities and at customer’s node is created randomly in the range of (1-8). Demand of each customer for each type of waste is known according to historical data. Mentioned model with described information will be solved by augmented $\epsilon$-constraint method; so some solutions will be obtained, and then between this Pareto optimal solutions, one solution with AHP technique will be selected. This model is solved by GAMS 24.3.3 and computations are done by CPLEX Solver. Computations had been done in a computer with 2.5 GHz two processors and WINDOWS 10.1 operating system is used as a technical platform.

Figure 3. Case study map

<table>
<thead>
<tr>
<th></th>
<th>Financial cost</th>
<th>Environmental cost</th>
<th>Social cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>502.99</td>
<td>312.66</td>
<td>801.54</td>
</tr>
<tr>
<td>2</td>
<td>513.66</td>
<td>299.88</td>
<td>788.27</td>
</tr>
<tr>
<td>3</td>
<td>545.11</td>
<td>335.85</td>
<td>731.25</td>
</tr>
<tr>
<td>4</td>
<td>555.47</td>
<td>345.51</td>
<td>690.24</td>
</tr>
<tr>
<td>5</td>
<td>550.65</td>
<td>340.24</td>
<td>720.89</td>
</tr>
<tr>
<td>6</td>
<td>558.69</td>
<td>328.87</td>
<td>682.75</td>
</tr>
<tr>
<td>7</td>
<td>540.23</td>
<td>361.24</td>
<td>739.66</td>
</tr>
<tr>
<td>8</td>
<td>530.55</td>
<td>399.54</td>
<td>756.86</td>
</tr>
<tr>
<td>9</td>
<td>565.98</td>
<td>361.36</td>
<td>670.55</td>
</tr>
<tr>
<td>10</td>
<td>523.65</td>
<td>360.78</td>
<td>781.69</td>
</tr>
</tbody>
</table>
Problem with time window=2 and 4 internal vehicles is solved by augmented epsilon constraint method and the results are reported in table 3 and figure 4. It is obvious from results that a trade-off between these three objectives should be done. In other words, the first objective function tries to reduce transportation costs. This goal can be achieved by using more with low level technology and it should be noted that this types of vehicle will release more pollutants to environment and caused to environmental costs. Because of importance of environment, each supply chain should try to reduce amount of CO2 which is released while the vehicles are servicing to customers. This goal is supported by means of the second objective function. The second objective function for reducing CO2 emission released during transportation time leads our network to use vehicles that have advanced technology and produce less pollutants during transportation. Moreover, for each customer a time window is considered. It means that arriving at customers' location earlier or later than specified time will lead to extra cost to the system. This cost is addressed in the third objective function in which we want to reduce deviation from specified servicing time by customers. In addition, each customer has different priority level, so this deviation will pose different cost values to the system with respect to corresponding customer. The results show that by increasing the amount of social cost objective functions, other objective functions are changed based on Pareto solution. So it can be concluded that three objective functions have trade-off with each other.

5.2 Sensitivity Analysis

Doing sensitivity analysis, decision makers can investigate effects of different parameters on the whole system.

For this purpose, in this study two types of sensitivity analysis are done. Initially this action is done on number of internal vehicles and in the second step sensitivity analysis is done on time window.

5.2.1 Number of Internal Vehicles Sensitivity Analysis

For sensitivity analysis on number of internal vehicles, the number of internal vehicles will be changed and its effects on objective functions
will be analyzed. For this analysis, eight levels of numbers are considered (number of internal vehicles=0, 1, 2, 3, 4, 5, 6, 7) and time window is 4 unit of time. Number of internal vehicles will affect each three objective functions. The result of sensitivity analysis is shown by table 4 and figure 5.

It is concluded from table 4 and figure 5 that as the number of internal vehicles increases, financial cost decreases, but after three vehicles, this relation is reversed. This trend shows the advantage of close-open mixed VRP against strict open or close VRP. The initial part of diagram is open VRP, and by going to the end of the diagram we are approaching to close VRP. It is obvious in initial and end part of diagram that the financial cost is more than the financial cost in middle part, and this middle part is COMVRP. Environmental cost will be increase constantly.

<table>
<thead>
<tr>
<th>Number of internal vehicles</th>
<th>Financial cost</th>
<th>Environmental cost</th>
<th>Social cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>656.48</td>
<td>270.65</td>
<td>598.35</td>
</tr>
<tr>
<td>1</td>
<td>611.24</td>
<td>284.56</td>
<td>609.46</td>
</tr>
<tr>
<td>2</td>
<td>584.68</td>
<td>302.47</td>
<td>621.84</td>
</tr>
<tr>
<td>3</td>
<td>550.07</td>
<td>314.15</td>
<td>635.94</td>
</tr>
<tr>
<td>4</td>
<td>591.49</td>
<td>338.84</td>
<td>647.33</td>
</tr>
<tr>
<td>5</td>
<td>624.09</td>
<td>352.64</td>
<td>666.41</td>
</tr>
<tr>
<td>6</td>
<td>667.89</td>
<td>378.95</td>
<td>678.51</td>
</tr>
<tr>
<td>7</td>
<td>701.89</td>
<td>395.12</td>
<td>690.08</td>
</tr>
</tbody>
</table>

Figure 5. Number of internal vehicles sensitivity analysis
The reason is that internal vehicles start their route at depot and end it at the depot, too. In other words, for servicing to first customer, internal vehicles should have a trip from depot to the first customer and after visiting last disposal facility they should have a trip from last disposal facility to the depot, but external vehicles do not have these two moves, so they will move less than internal vehicles and they will emit less CO2 in comparison with internal vehicles. Because of the first excessive trip of internal vehicles in comparison to external ones, so internal vehicles often arrive at customer nodes later than externals and this lateness leads to more penalty in comparison with arriving soon. As such by increasing in the number of internal vehicles social cost will be increased.

5.2.2 Time Window Sensitivity Analysis

In this section, the time window limitation which is determined for each customer will be gradually changed. The time between earliest time at customer’s location and latest time is selected for sensitivity analysis. Eight levels for this time window are considered (time window=1 through 8 minutes) and number of internal vehicles is assumed to be three. Time limitation and penalty of violation from this limitation affect the each three objective function in different manner. If this limitation decreases, it means that vehicles should arrive at customer locations earlier; therefore, in order to avoid more penalty, system may decide to establish more vehicles and this decision will result in the first and second objective functions. Since the cost of establishing new vehicles increases, it will affect the first objective value. The second objective function is also affected because, in order to avoid more social cost, more vehicles should serve customers simultaneously, and because each vehicle should visit all disposal facilities, more distance will be passed and more CO2 will be emitted. All in all, this sensitivity analysis showed that by decreasing and increasing time limitation; each three objective functions will be affected. Table 5 shows the results of this experiments.

<table>
<thead>
<tr>
<th>Time between earliest and latest at each customer node</th>
<th>Financial cost</th>
<th>Environmental cost</th>
<th>Social cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>641.88</td>
<td>421.25</td>
<td>754.25</td>
</tr>
<tr>
<td>2</td>
<td>613.55</td>
<td>373.56</td>
<td>720.45</td>
</tr>
<tr>
<td>3</td>
<td>588.86</td>
<td>339.57</td>
<td>690.24</td>
</tr>
<tr>
<td>4</td>
<td>550.07</td>
<td>314.15</td>
<td>635.94</td>
</tr>
<tr>
<td>5</td>
<td>535.49</td>
<td>298.98</td>
<td>611.89</td>
</tr>
<tr>
<td>6</td>
<td>526.45</td>
<td>290.64</td>
<td>601.35</td>
</tr>
<tr>
<td>7</td>
<td>524.24</td>
<td>289.48</td>
<td>599.45</td>
</tr>
<tr>
<td>8</td>
<td>523.94</td>
<td>287.65</td>
<td>598.55</td>
</tr>
</tbody>
</table>
It can be concluded from table 5 and figure 6 that by increasing the time between earliest and latest time at each customer node, the financial, environmental and social cost will be decreased. The reason is that by this action, the need for establishing new vehicles for servicing will decrease. It should be noted that after a while there is no significant changes in financial, environmental and social costs by increasing amount of time window.

6. Discussion and Conclusion

A sustainable vehicle routing problem was discussed in this paper. In the proposed model, in order to reach a sustainable VRP, three kinds of objective functions were considered. The first one was financial costs and it was related to minimizing transportation costs. The second objective function was about environmental costs and it tried to achieve this goal by minimizing amount of CO₂ emitted by vehicles during transportation and servicing. The last one was about social costs. Third objective function attempted to pay more attention to customer by arriving and leaving customer’s locations at determined time. Both arriving soon or late at customer’s locations pose cost to system and the third objective incorporates this issue by minimizing violation from time window tries in order to gain high level of customer’s satisfaction. It should be noted that in this model a weight was considered for each customer and customers with more weight had more importance for the system. Additionally, in this paper, heterogeneous fleet of vehicles was considered; that is, each type of vehicle has different capacity, time limitation, fixed and variable costs. The routes were considered mixed open and close and multi depot was used for modeling the problem. Several types of wastes were considered and each type of waste should be unloaded at appropriate disposal facilities. The application of proposed model was investigated in a real case study and augmented epsilon constraint and CPLEX solver were used for solving the problem of this case and validating the proposed model. By analyzing the results it is concluded that three types of objective functions have a conflict among themselves. In the next step, sensitivity analysis was done. This analysis was about the effects of two important parameters including number of internal vehicles and ranges of time windows on the objective functions. It is concluded that by increasing number of internal vehicles the first objective function will decrease initially and after a while it will increase. Moreover, the second and third objective...
functions will increase constantly by increasing number of internal vehicles. In the second sensitivity analysis it is concluded that by decreasing the range of time limitation for reducing the cost of violation from time window, system should apply more vehicles. This action poses more financial cost to model and by applying more vehicles for servicing to customers more distances should be passed and more CO2 will be emitted.

For future researches it is proposed that uncertainty on time between nodes and the demand of customer can be considered and some meta-heuristic approaches can be developed to solve large scale problems.

7. References


Sustainable Vehicle-Routing Problem with Time Windows by Heterogeneous Fleet of Vehicles ...


Sustainable Vehicle-Routing Problem with Time Windows by Heterogeneous Fleet of Vehicles


