Nonlinear Stabilizing Controller for a Special Class of Single Link Flexible Joint Robots

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Abstract

Joint flexibility is a very important factor to consider in the controller design for robot manipulators if high performance is expected. Most of the research works on control of flexible-joint robots in literature have ignored the actuator dynamics to avoid complexity in controller design. The problem of designing nonlinear controller for a class of single-link flexible-joint robot manipulators whose model incorporates the effect of the electrical actuator is considered in this paper. The main control purpose followed in this research is stabilization of the system states and backstepping approach is employed to achieve this goal and find control law. The global asymptotic stabilization of the closed-loop system is achieved in the sense of Lyapunov. Finally, to demonstrate the efficiency of the designed controller in stabilization the system states, the simulation results for system dynamics and closed-loop system, are compared in different initial conditions without and in the presence of external disturbances.

Keywords: Nonlinear Controller, Flexible joint manipulator, Stabilization, Electrical actuator.

1. Introduction

In the recent years, considerable research efforts have been made to solve the problem of flexible-joint robot control. Joint flexibility exists when there is a difference between the angular position of the driving actuator and that of the driven link. It is known that the joint flexibility can cause oscillations in robot manipulators. Therefore, it is considered as a problem. Good et al. [1] showed that ignoring joint flexibility in manipulator dynamics and controller design causes degradation in performance of robots.

A flexible joint robot (FJR) model was introduced by Spong [2], which led to many researches on flexible joint robot manipulators. Several approaches have been so far suggested for controller design of FJRs: PD control [3], feedback linearization [4], singular perturbation approach [5], extended state observer based control [6], using neural networks [7] and fuzzy logic controller [8], to cite a few. A survey of the literature related to modelling and control of flexible joint robots is provided in [9].

However, in all the aforementioned studies, the dynamics of the actuator has been neglected in modelling and controller design. The main reason is that considering the actuator dynamics will cause more complexity in controller design. There are very few contributions in the literature about FJRs in which actuator dynamics is considered in dynamic model and controller design. Iterative regulation of an electrically driven flexible-joint robot was presented in [10]. A linear output-based controller for stabilization of a flexible- joint electrically driven robot is investigated in [11]. A simple adaptive robust control structure is designed for an Electrical flexible joint robot manipulator under both structured and unstructured uncertainty in [12]. A robust tracking control is suggested for a class of electrically driven FJRs actuated by brushed DC motors[13]. The control aims were accomplished without velocity measurements via designing a nonlinear observer.

Backstepping approach is suggested as a method providinga framework for recursive design of nonlinear systems by achieving system stability in each step[14],[15]. Backstepping is applied to stabilize the dynamic motion of the FJR [16]. Robust backstepping control to overcome parameter uncertainty for FJRs is proposed in[17].

In this paper backstepping is employed for nonlinear controller design of a class of single-link flexible-joint robots incorporating the dynamics of the brushed DC motor used as actuator, in the dynamic model. The control purpose is to stabilize the system states in a few seconds considering different initial conditions. First, the dynamic model of the

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FJR incorporating motor dynamics is introduced briefly. Next the nonlinear controller design is done using backstepping approach. Then the stability analysis is studied for the closed-loop system and the global asymptotic stability is achieved. Finally, comparing the simulation results for the system states before and after designing controller in different initial conditions, the effectiveness of the proposed controller in stabilizing system states is verified. The effect of external disturbances on system states and control input is studied through simulation results, too.

2. Dynamics of FJR Actuated by Brushed dc Motor

The dynamic model of a rigid-link flexible joint manipulator can be derived from Lagrange equations [2] involving actuator dynamics of brushed DC motors, the governing dynamic equation of the single-link electrically driven flexible-joint robot which is shown in Fig. 1, is obtained easily as follows:

$$\begin{aligned} \ddot{\theta} &= \frac{-mgl}{I}\sin(\theta) - \frac{C}{I}\dot{\theta} - \frac{K}{I}(\theta - \theta_m) \\ \ddot{\theta}_m &= \frac{K}{J}(\theta - \theta_m) - \frac{B}{J}\dot{\theta}_m + \frac{K_m}{J}I_a \\ \dot{I}_a &= -\frac{K_b}{L}\dot{\theta}_m - \frac{R}{L}I_a + \frac{u}{L} \end{aligned} \quad (1)$$

θ, θ_m	joint and motor angles,
$\dot{\theta}$, $\dot{\theta}_m$	joint and motor velocities,
I, J	link and motor inertia coefficients,
C, B	joint and motor damping coefficients,
m, l	mass and length of the link,
K	joint stiffness coefficient,
K_m	motor torque constant,
K_b	back-emf constant,
R, L	armature resistance and inductance,
I_a	motor current,
U	motor voltage.

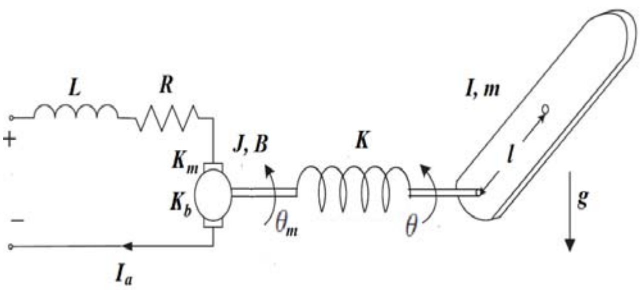


Fig. 1. Single-link flexible-joint robot manipulator actuated by brushed DC motor [13].

In the above modelling of the electrical flexible joint robot, joint flexibility is modelled by a linear torsional spring. The equation of motion of the electrical flexible joint robot is represented in the state-space in Eq. (2). The state vector contains angular position and angular velocity of the link side, angular position and angular velocity on the motor side and the motor current, i.e. $[x_1;x_2;x_3;x_4;x_5] = [\theta;\dot{\theta};\theta_m;\dot{\theta}_m;I_a]$

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{-mgl}{I}\sin(x_1) - \frac{C}{I}x_2 - \frac{K}{I}(x_1 - x_3) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{K}{J}(x_1 - x_3) - \frac{B}{J}x_4 + \frac{K_m}{J}x_5 \\ \dot{x}_5 &= -\frac{K_b}{L}x_4 - \frac{R}{L}x_5 + \frac{u}{L} \end{aligned} \quad (2)$$

As shown in the above system, the dynamic equation is very complex and highly nonlinear and controller design for this system is a serious challenge. In the next section, a control structure will be designed based on the backstepping approach to achieve the control purpose of stabilization of the system.

3. Backstepping Controller Design

The control method used for stabilization of the flexible-joint robot incorporating brushed DC motor dynamics is based on backstepping technique. The backstepping technique provides a recursive process for stabilizing of a system which must be in a form which is called strict-feedback form. Since the nonlinear equation of dynamic model represented in Eq. (2) is strict-feedback, backstepping controller design is a proper choice to accomplish our control purposes.

In general, the system which is to be controlled is given below:

$$\begin{aligned} \dot{x} &= f(x) + g(x)\xi \\ \dot{\xi} &= a(x,\xi) + b(x,\xi)u, \end{aligned} \quad (3)$$

Where $x \in R^n$ and $\xi \in R$ are state variables and $u \in R$ is the control input. First ξ is considered as a control input for the x-subsystem. ξ can be selected in any way to make the x-subsystem globally asymptotically stable. The choice is denoted $\xi^{des}(x)$ and is named a *virtual control law*. This virtual control law is determined from selection of a proper Lyapunov function V.

The order of system of Eq. (2), is 5. So, five proper Lyapunov functions and virtual control laws which are V_i and ξ_i^{des} for i=1,2,…,5 should be calculated in five steps and then the actual control input u is designed. Using some mathematical operations, these Lyapunov functions and virtual control laws are determined as (4).

$$\begin{cases} \xi_1^{des} = -c_1 x_1, \\ \xi_2^{des} = -x_1 + \dot{\xi}_1^{des} - k_1\left(\xi_1 - \xi_1^{des}\right), \\ \xi_{i+2}^{des} = \dot{\xi}_{i+1}^{des} - \left(\xi_i - \xi_i^{des}\right) - k_{i+1}\left(\xi_{i+1} - \xi_{i+1}^{des}\right), \quad i = 1,2,3 \end{cases}$$
$$\begin{cases} V_1 = \frac{1}{2}x_1^2, \\ V_{j+1} = \frac{1}{2}x_1^2 + \sum_{k=1}^{j}\frac{1}{2}\left(\xi_k - \xi_k^{des}\right)^2, \quad j = 1,2,3,4 \end{cases} \tag{4}$$

Where the new states ξ_i are achieved from the states x_i and the constants c_1, k_1, k_2 , k_3 , k_4 ,are the controller gains which should be positive. Above selections in Eq. (4) guarantees the global asymptotic stabilization of the closed-loop subsystems in the sense of Lyapunov.

Regarding to the state-space form in Eq. (2), the virtual control laws which achieve the global asymptotic stability of the closed-loop system of the FJR incorporating motor dynamics in the dynamic model, are obtained as follows:

$$\begin{aligned} \xi_1^{des} &= -c_1 x_1, \\ \xi_2^{des} &= -c_1 x_2 - x_1 - k_1\left(\xi_1 - \xi_1^{des}\right), \\ \xi_3^{des} &= c_1\left(\frac{-mgl}{I}\sin(x_1) - \frac{C}{I}x_2 - \frac{K}{I}(x_1 - x_3)\right) - x_2 \\ &\quad - k_1\left(\xi_2 + c_1 x_2\right) - \left(\xi_1 - \xi_1^{des}\right) - k_2\left(\xi_2 - \xi_2^{des}\right) \end{aligned} \tag{5}$$

The forth virtual control law is determined in fourth step of backstepping controller design as follows:

$$\begin{aligned} \xi_4^{des} &= c_1\left(\frac{-mgl}{I}x_2\cos(x_1) - \frac{C}{I}\left(\frac{-mgl}{I}\sin(x_1) - \frac{C}{I}x_2 - \frac{K}{I}(x_1 - x_3)\right)\right) \\ &\quad - c_1\frac{K}{I}(x_2 - x_4) - \left(\frac{-mgl}{I}\sin(x_1) - \frac{C}{I}x_2 - \frac{K}{I}(x_1 - x_3)\right) \\ &\quad - k_1\left(\xi_3 + c_1\left(\frac{-mgl}{I}\sin(x_1) - \frac{C}{I}x_2 - \frac{K}{I}(x_1 - x_3)\right)\right) \\ &\quad - \left(\xi_2 + c_1 x_2\right) - k_2\left(\xi_3 - c_1\left(\frac{-mgl}{I}\sin(x_1) - \frac{C}{I}x_2 - \frac{K}{I}(x_1 - x_3)\right)\right) \\ &\quad + k_2\left(-x_2 - k_1\left(\xi_2 + c_1 x_2\right)\right) - \left(\xi_2 - \xi_2^{des}\right) - k_3\left(\xi_3 - \xi_3^{des}\right) \end{aligned} \tag{6}$$

Also, the last virtual control law, ξ_5^{des}; is obtained by some mathematical operations from Eq. (4). In Eq. (5) and Eq. (6), the virtual states ξ_i for i=1,2,3 are determined from the system state-space equation as:

$$\begin{aligned} \xi_1 &= x_2, \\ \xi_2 &= \frac{-mgl}{I}\sin(x_1) - \frac{C}{I}x_2 - \frac{K}{I}(x_1 - x_3), \\ \xi_3 &= -\frac{mgl}{I}x_2\cos(x_1) - \frac{K}{I}x_2 + \frac{K}{I}x_4 \\ &\quad - \frac{C}{I}\left(\frac{-mgl}{I}\sin(x_1) - \frac{C}{I}x_2 - \frac{K}{I}(x_1 - x_3)\right). \end{aligned} \tag{7}$$

Finally, the actual feedback-stabilizing control law u is derived as:

$$\begin{aligned} u &= \frac{IJL}{KK_m}\left(\xi_5^{des} + \frac{mgl}{I}\left[-\frac{mgl}{I}x_2\cos(x_1) - \frac{C}{I}\dot{x}_2 - \frac{K}{I}(x_2 - x_4)\right]\cos(x_1)\right) \\ &\quad + \frac{JLC}{KK_m}\left(-\left(\frac{2K}{I} - \frac{C^2}{I^2}\right)\dot{x}_2 + \frac{K}{I}\dot{x}_4 - \frac{C}{I}\left(\frac{-mgl}{I}x_2\cos(x_1) - \frac{K}{I}(x_2 - x_4)\right)\right) \\ &\quad + \frac{IJL}{KK_m}\left(-\frac{mgl}{I}x_2^3\cos(x_1) - \frac{3mgl}{I}x_2\dot{x}_2\sin(x_1)\right) \\ &\quad - \frac{JL}{K_m}\left(-\frac{mgl}{I}x_2\cos(x_1) - \frac{K}{I}(x_2 - x_4)\right) + \frac{LB}{K_m}\dot{x}_4 + K_b x_4 + R x_5 \end{aligned} \tag{8}$$

Where $\dot{x}_2$ and $\dot{x}_4$ are substituted from Eq. (2). Based on the backstepping approach, it can be concluded that the designed control law represented in Eq. (8) guarantees the global asymptotic stability of the closed-loop system of governing Eq. (1) and Eq. (2).

4. Numerical Simulations

To illustrate the effectiveness of the suggested nonlinear backstepping controller, numerical simulations are performed in this section. The values of parameters of the dynamic model of EFJR, are demonstrated in table 1. The controller gains are selected as: c_1=4, k_1 =5, k_2 =4, k_3 =5 and k_4 =4by trial and error. Values of parameters in numerical simulations are:

Mgl=5(Nm),K=100(Nm/rad),I=1(Kg.m^2),J=0.3(Kg. m^2),C= 0.02(Nm.s/rad),B=0.001(Nm.s/rad),K_m=0.2(Nm/A),K_b=0.26(Nm/A),R= 1.6(Ω), L=0.01(H).

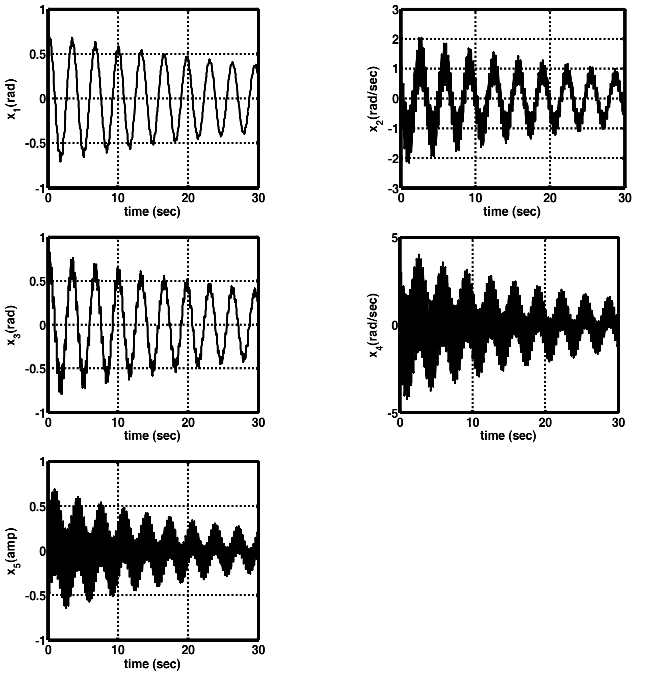
in this section, simulations comparisons before and after designing the controller for the system, show the efficiency of the proposed controller in stabilization of the system states in different initial conditions. Then the effect of external disturbances on system states in different initial conditions studied. For all of the simulation cases, the mentioned values of the system parameters are used.

a. Case1

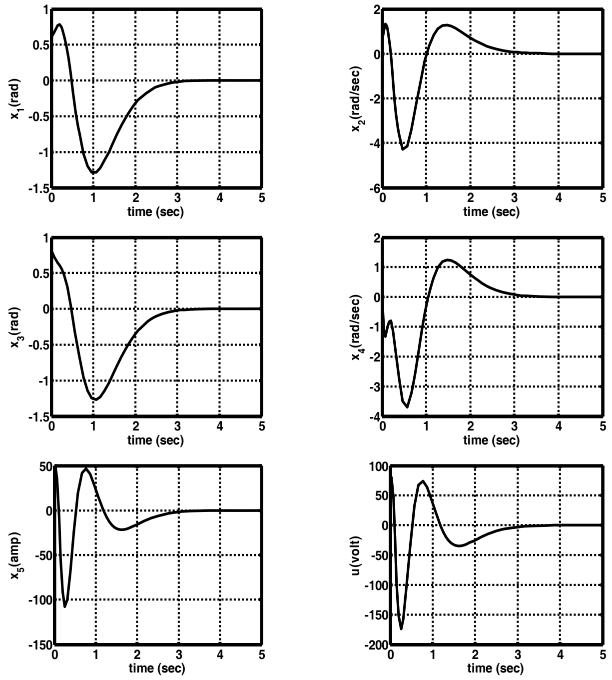
The Initial conditions of the system states in this simulation are chosen as [0.6; 0.5; 0.8; 0.3; 0]. Simulation results in Fig. 2(a) represent the variations of the system states before designing the controller. These states are joint angle (x_1), joint velocity (x_2), motor angle (x_3), motor velocity (x_4) and motor current (x_5), respectively. As shown in Fig. 2(a), the dynamic of the system has a very oscillatory behavior and the settling time is too long; even after 30 Sec., the system states has not been stable. After designing the controller for the system, all of the system states are stabilized and converge to equilibrium point [0,0,0,0,0], in 3 seconds. Fig. 2(b) illustrates the effect of the proposed controller on the stabilization of the system states. The input control signal is motor voltage and it is shown in fig.2(b), too.

b. Case2

In this case initial conditions of the system states are chosen as [0.8; 0.5; 0.5; 0.2; 0]. As shown in Fig. 3 the proposed controller has a good performance in stabilization of

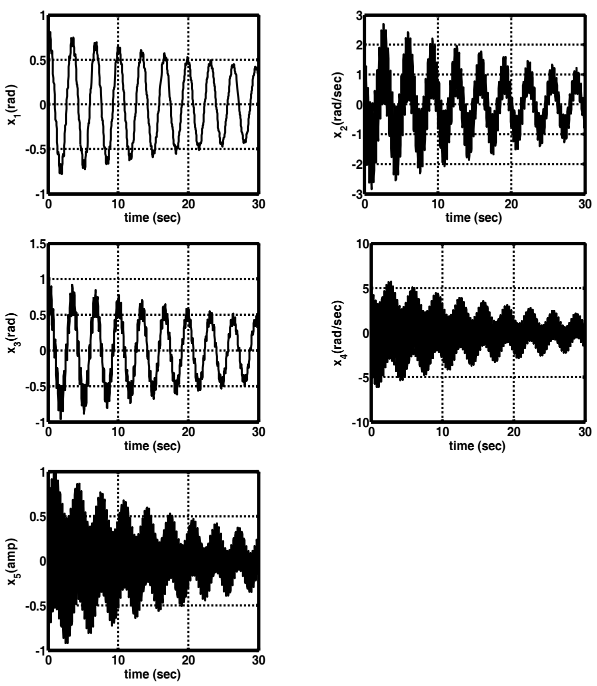


(a) The system states before stabilization

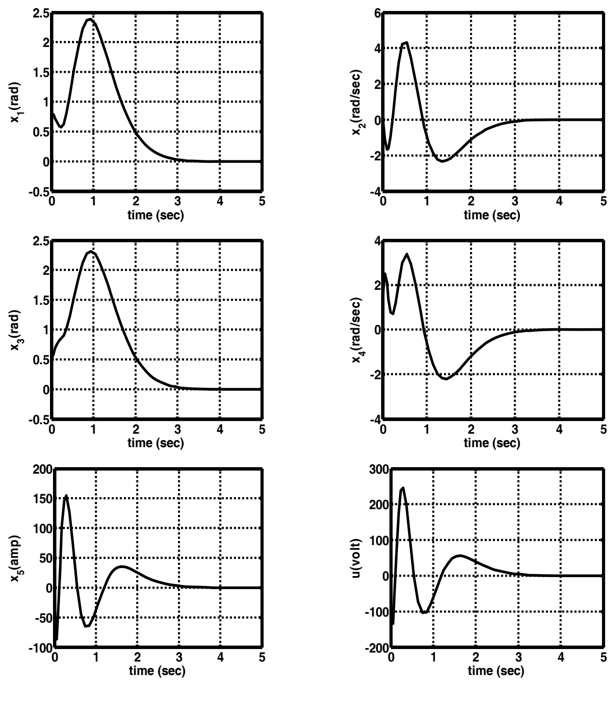


(b) The system sates after stabilization and controller input

Fig. 2.The effect of controller design in the stabilizing of the system states in case1



(a)The system states before stabilization



(b) The system sates after stabilization and controller input

Fig3. The effect of controller design in stabilizing of the system states in case2 of the system states and all of the states are converged to equilibrium point[0;0;0;0;0], in less than 3 seconds.

Comparing the results in each case, we find out that the suggested controller has caused the elimination of the oscillations and considerable reduction in settling time. All of the states of the closed-loop system converge to the equilibrium point [0;0;0;0;0] in few seconds.

c. Case3

In this case, the effects due to external disturbances into the closed-loop system behavior are considered via simulation results to show the robustness of the suggested controller.

Two unknown disturbances d_1 and d_2 are added into the joint angle and motor angle 2nd-order dynamics. These two signals have random values and occur between t=1.5 *sec* and t=2.5 *sec*. Simulation results for stabilization of the system are represented in Fig. 4. Initial conditions for this simulation case are the same as the simulation in subsection A. Fig.4 shows the good robustness of the controller in stabilization in presence of external disturbances.

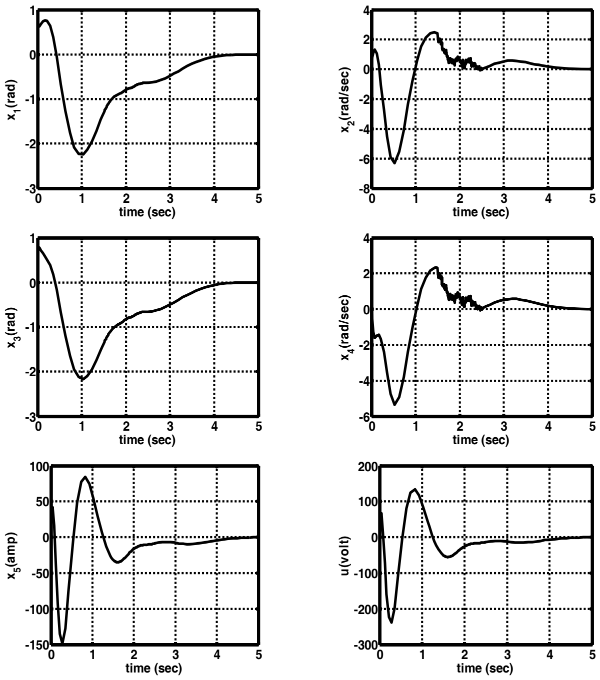


Fig. 4. The effect of external disturbances on system states and control input in case1

Similarly, the effects of the external disturbances on the controller performance in stabilization of the system states are studied via simulation results considering the initial conditions of subsection B. The results are shown in Fig.5 and depicted

the acceptable performance of the designed controller in stabilizing the system states under external disturbances (which are happened between *t=1.5 sec* and *t=2.5 sec*).

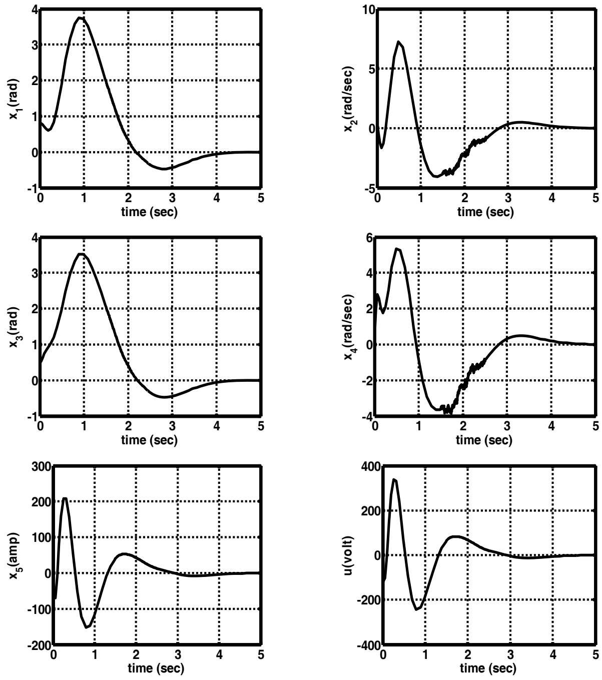


Fig. 5.The effect of external disturbances on system states and control input in case2

5. Conclusion

In this paper, a nonlinear controller for a single-link flexible-joint robot manipulator whose model incorporates the effects of the brushed DC motor has been established. To accomplish this control purpose, a nonlinear controller was designed based on backstepping approach and asymptotic global stability of system states has been achieved. Comparing the simulation results before and after controller design, it was concluded that the proposed nonlinear controller has a good performance in stabilization the system states in different initial conditions. To show the robustness of the suggested controller, external disturbances are added into the joint angle and motor angle 2nd-order dynamics and simulation results illustrate the acceptable performance of the designed controller in the presence of the external disturbances. The advantage of the proposed control scheme is that it does not need measurement of acceleration and jerk of motor side and link side.

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