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Research Article

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Calculating Cost Efficiency with Integer Data in the Absence of Convexity

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Abstract

One of the new topics in DEA is the data with integer values. In DEA classic models, it is assumed that input and output variables have real values. However, in many cases, some inputs or outputs can have integer values. Measuring cost efficiency is another method to evaluate the performance and assess the capabilities of a single decision-making unit for manufacturing current products at a minimum cost with its input prices. In this paper, we proposed a model which is capable of calculating the cost efficiency in the absence of convexity when some of the input parameters have integer values, and then we implemented the mentioned model with a numerical example and discussed the results.

Keywords: Data envelopment analysis (DEA), Cost efficiency, Integer data, Convexity.

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1. Introduction

DEA is a technique based on mathematical programming for evaluating the performance of a set of congruent decision making units (DMUs) with several inputs and outputs. In fact, DEA identifies organizational strengths and weaknesses regarding each index. DMUs can refer to the hospitals, universities, etc. DEA is a powerful tool which is now widely used in evaluating the performance of systems with several inputs and outputs. DEA was proposed in 1973-1979 by Charnes and Cooper [1]. They based their work on Farrell's research [2]. Results of this investigation was the CCR article. Following that, Banker, Charnes, and Cooper proposed BCC article [3].

Cost efficiency evaluates the ability of a decision-making unit to manufacture current products at minimum cost. Convex cost efficiency (CCE) proposed by Camanho and Dyson [4] as an output or input adjusted cost efficiency evaluation method has several interesting features.

Camanho and Dyson [4] stated that the scales of inputs andoutputs are simultaneously adjusted based on a fixed levelof income. It might also enhance competitive nature of the market (use of a utility function becomes necessary when the input and output objectives are simultaneously obtained with regard to the input and output prices). However, the observed inputs and outputs can be significantly different from objective inputs and outputs in maximizing the profits. This is because profit maximization simultaneously needs to minimize costs and maximize income but cost efficiency only requires to minimize costs of the current income of the evaluated DMU.

In DMU models, it is assumed that input and output variables have real values. However, in many cases in the real world, some inputs or outputs can only have integer values. Them.

In 2006, Villa-Lozano [5] showed the model with integer data in DEA for the first time by offering a linear program (MILP) which limited the calculated objectives in integers. Then, we examined the second part of cost efficiency in terms of returns to scale (RTS). Camanho and Dyson [4] introduced the CCE scale based on DEA-CRS technology. In the third part, we studied the alternative model proposed by Fukuyama et al. [6] for cost efficiency which investigates the alternative form of radial CCE measurement. In Part four, we discuss the adjustment model of non-convex cost efficiency in terms of RTS; and the expanded model of Fukuyama [6] explaining the CCE method with an NCCE non-convex cost efficient method is studied. In Part five, we proposed a model with integer inputs. In part six, we offered a model for calculating cost efficiency with the use of integer data and also proposed a numerical example in line with that model. Finally, we presented a summary of results.

2. Convex cost efficiency in terms of fixed returns to scale

Suppose that c_{nj} is the input price of n and r_{mj} is the output price of m for DMU_j and suppose that all prices are totally clear. Camanho et al. [4] defined CCE scale based on DEA- CRS technology as follows:

$$CCE^{camanho\text{-}dyson} = \frac{1}{\alpha_0} \times min \sum_{n=1}^{N} c_{no}\, x_n$$

$$\sum_{j=1}^{J} \lambda_j y_{mj} \geq y_m \qquad \forall m$$

$$\sum_{j=1}^{J} \lambda_j y_{nj} \leq x_n \qquad \forall n$$

$$\sum_{m=1}^{M} r_{mo}\, y_m = \xi$$

$$x_n \geq 0 \qquad \forall n \qquad (1)$$

$$y_m \geq 0 \qquad \forall m$$

Here and are respectively the total cost and total revenue. Fukuyama et al. [6] respectively refer to the model (1) and the optimal value as the CCE model of Camanho-Dyson [4] and returns to scale of Camanho-Dyson [4].

If $\sum_{m=1}^{M} r_{mo}\, y_m = \varepsilon$ is replaced in model (1) by $\sum_{m=1}^{M} r_{mo}\, y_m = \varepsilon$, cost efficiency is provided by Fir et al. [7]. Camanho and Dyson [4] stated that the current income is shown in the prices and values of DMU_o, *i.e.* $\xi_0 = \sum_{m=1}^{M} r_{mo}\, y_{mo}$ Hence, $CCE^{camanho\text{-}dyson}$ has a theoretical basis in Shepherd's indirect cost function.

3. The alternative model for cost efficiency

The following model is somewhat equivalent to the model (1) and the measurements shown by the relevant real functions are equal. Fukuyama et al. proposed the alternative form of radial CCE which is similar to the radial DEA measurement.

$$CCE_0^{radial} = \min \theta$$

$$s.t \quad \sum_{j=1}^{J} \lambda_j \left(\sum_{n=1}^{N} c_{no}\, x_{nj}\right) \leq \theta\alpha_0$$

$$\sum_{j=1}^{J} \lambda_j y_{mj} \geq y_m \qquad \forall m$$

$$\sum_{m=1}^{M} r_{mo}\, y_m = \xi$$

$$\theta = \text{free} \qquad \lambda_j \geq 0 \qquad \forall j \qquad (2)$$

$$y_m \geq 0 \qquad \forall m$$

Fukuyama [6] stated that by calculating the total cost and revenue for all DMUs from the input and output prices in the total evaluated DMU, we obtain the sum of revenues used as a general input and output. Model (2) can be converted to the following model:

$$CCE_0^{radial} = \min \theta$$

$$s.t \quad \sum_{j=1}^{J} \lambda_j \left(\sum_{n=1}^{N} c_{no}\, x_{nj}\right) \leq \theta\alpha_0$$

$$\sum_{j=1}^{J} \lambda_j \sum_{m=1}^{M} r_{mo}\, y_{mj} \geq \xi$$

$$\theta = \text{free} \qquad , \lambda_j \geq 0 \qquad \forall j \qquad (3)$$

Model (3) has two constraints and the selected variables of j + Measurement in CCE scale of Camanho-Dyson [4] supposes that the evaluated DMU defines the inputs and outputs and the objective. On the other hand, the radial CEE of the ratio of total cost defined by the evaluated DMU to the total cost observed in the prices of DMU minimizes the inputs and outputs.

4. The extended model of Fukuyama non-convex cost efficiency

Fukuyam generalized model [6] extending the

CCE method toa proper method of cost efficiency with non-convex cost effectiveness is as follows:

$$\frac{1}{\alpha_0} \times \min \sum_{n=1}^{N} c_{no}\, x_n$$

$$s.t \qquad \sum_{j=1}^{J} \lambda_j\, y_{mj} \geq y_m \qquad \forall m$$

$$\sum_{j=1}^{J} \lambda_j\, y_{nj} \leq x_n \qquad \forall n$$

$$\sum_{m=1}^{M} r_{mo}\, y_m = \xi_0$$

$$\lambda_j \epsilon \{0,1\} \qquad \forall j \qquad (4)$$

$$x_n \geq 0 \qquad \forall n, \quad y_m \geq 0 \qquad \forall m$$

5. Input and output indexes with integer values

In DEA models, it is assumed that input and output variables have real values. However, in many real management cases, some inputs or outputs can only have integer values. In 2006, Vila and Lozano [5] showed the model with integer data in DEA for the first time by offering a model of mixed integer linear programming (MILP) which limited calculation targets to integers.

In some cases, rounding up the DEA answer to the closest integers can lead to the wrong diagnosis of efficiency and executive targets. Rounding up executive targets to the nearest integer does not necessarily lead to a big difference for larger parts but is an important issue in small parts with small input and output scales.

Given that we face integer data in many practical applications, extending DEA classic models for conditions where some inputs and outputs have integer values is necessary. Many studies have been done in this field. The principles proposed by Kazemi Matin and Kusmanen [8] in DEA with integer data are as follows:

1) Integer principle: If (x,y) ∈ T, then $(\boldsymbol{x}, \boldsymbol{y}) \in \mathbb{Z}^{\boldsymbol{m}+\boldsymbol{s}}$

2) The principle of natural convexity: If $(x_1, y_1) \in$ T and $(x_2, y_2) \in$ T, then for each λ∈[0,1], we will have

$\lambda\,(x_1, y_1)+(1-\lambda)\,(x_2, y_2) \in \mathbb{Z}^{\boldsymbol{m}+\boldsymbol{s}}$

$\lambda\,(x_1, y_1)+(1-\lambda)\,(x_2, y_2) \in$ T

The concept of natural convexity resulting from the convexity principle is a state where the convex combination of two feasible production plans offering an input and output vector with integer value.

6. The proposed model to calculate efficiency with the use of integer data in the absence of convexity

If x_j is the input; y_j is the output; *c* is the cost vector, and cx_p is the cost level of DMU_p, we present the following model for evaluation of cost efficiency in the absence of convexity: (5)

Min θ

$$s.t \quad \sum_{j=1}^{n} \lambda_j\, x_j = \theta c x_p$$

$$\sum_{j=1}^{J} \lambda_j\, y_j \geq y_p \qquad r = 1, \ldots, n$$

$$\lambda_j \epsilon \{0,1\}$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

If the inputs have the integer condition, we can develop model (5) as the following which can investigate cost efficiency with integer data despite the absence of convexity:

Min θ

$$s.t \quad \sum_{j=1}^{n} \lambda_j cx_j = \theta cx_p$$

$$\sum_{j=1}^{n} \lambda_j cx_j = z_i$$

$$\sum_{j=1}^{n} \lambda_j y_j \geq y_p \qquad r = 1, \dots, n$$

$$\lambda_j \epsilon \{0,1\},$$

$$\sum_{j=1}^{n} \lambda_j = 1 \quad , z_i = integer$$

Now, if we calculate θ from the first constraint and replace it in the objective function, the model is converted as follows:

$$\text{Min} \frac{1}{cx_{\text{p}}} \times \sum_{j=1}^{n} cx_j$$

$$\sum_{j=1}^{n} \lambda_j cx_j = z_i$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rp}$$

$$\lambda_j \epsilon \{0,1\},$$

$$\sum_{j=1}^{n} \lambda_j = 1 \quad , z_i = integer$$

7. Numerical example

We assume that we have six DMUs with three inputs and one output as the following table:

Table 1: Data table.

	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6
I_1	3	1	4	2	5	1
I_2	2	3	6	5	3	2
I_3	4	2	5	1	7	6
O_1	3	5	6	3	1	4

By running the model (7) with input and output values of table (1) in GAMS, we obtain data as table (2). Here, s (i) indicatesthe constraints, w indicates cx_j and cost efficiency shows the cost efficiency on integers.

According to Table 2, we see that DMU_2 is the cost efficiency and other DMUs are cost inefficient. In other words, the amount of cost DMU_2 spends to produce its output is 20 units which means that DMU_2 has produced its output with the lowest possible cost and is thus cost efficient. In terms of efficiency, DMU_2 is ranked first and DMU_4, DMU_3, DMU_1, DMU_6, and DMU_5 are ranked second to sixth.

8. Conclusion

We began with an introduction on the analysis of DEA and investigated the convex and non-convex cost efficiency in terms of returns to scale with CCE model of Camanho and Dyson. We also offered an alternative model for improving cost efficiency. After reviewing efficiency calculation models, we proposed a model which is able to calculate cost efficiency with integer data. One of the DMUs was cost integer data. One of the DMUs was cost efficient and the others were cost inefficient.

Table 2: Cost efficiency.

	$s(i_1)$	$s(i_2)$	$s(i_3)$	w	Cost - efficiency
DMU_1	1	2	2	17	0.548
DMU_2	1	3	2	20	1.000
DMU_3	2	4	3	28	0.609
DMU_4	1	2	2	17	0.944
DMU_5	1	1	1	9	0.170
DMU_6	1	3	2	19	0.463

data. One of the DMUs was cost efficient and the others were cost inefficient.

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