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Zagreb Indices and Coindices of Total Graph, Semi-Total Point Graph and Semi-Total Line Graph of Subdivision Graphs

Harishchandra S. Ramane ⋆, *Saroja Y. Talwar and Ivan Gutman*

Abstract

Expressions for the Zagreb indices and coindices of the total graph, semi-total point graph and of semi-total line graph of subdivision graphs in terms of the parameters of the parent graph are obtained, thus generalizing earlier existing results.

Keywords: Zagreb indices, Zagreb coindices, total graph, semi-total point graph, semi-total line graph, subdivision graph.

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1. Introduction

Topological indices are numerical quantities associated with a graph that are invariant under graph isomorphism. The interest in topological indices is due to their applicability in quantitative structure–property relationship (QSPR) and quantitative structure–activity relationship (QSAR) studies in chemistry [3, 12]. Many of these topological indices are based on degrees of vertices [5].

Let G be a simple graph with the vertex set $V(G)$ and the edge set $E(G)$. The number of vertices and number of edges of G will be denoted by n and m, respectively. The edge connecting the vertices u and v is denoted by uv. The

⋆Corresponding author (E-mail: hsramane@yahoo.com)
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degree of a vertex v, denoted by $d_G(v)$, is the number of edges incident to it in G. The first and second Zagreb index of G are defined as

$$M_1(G) = \sum_{u \in V(G)} d_G(u)^2 = \sum_{uv \in E(G)} \big[d_G(u) + d_G(v)\big]$$

and

$$M_2(G) = \sum_{uv \in E(G)} d_G(u)\, d_G(v)\,.$$

The Zagreb indices belong among the oldest degree–based molecular structure descriptors and have been extensively studied. For details see the recent review [2] and the references cited therein.

The first and second Zagreb coindices are defined as [1, 4]

$$\overline{M_1}(G) = \sum_{\substack{uv \notin E(G) \\ u \neq v}} \big[d_G(u) + d_G(v)\big] \qquad \text{and} \qquad \overline{M_2}(G) = \sum_{\substack{uv \notin E(G) \\ u \neq v}} d_G(u)\, d_G(v)\,.$$

Details of their mathematical properties can be found in the survey [6].

Ranjini et al. [10] calculated the Zagreb indices and coindices of the line graph of the subdivision graph of tadpole graph, wheel and ladder. Later Ramane et al. [9] generalized these results by finding the Zagreb indices of the line graph of the subdivision graph of any graph.

In [8], Mohanappriya and Vijayalakshmi obtained the Zagreb indices of the total graph of the subdivision graph of tadpole graph, wheel and ladder.

In this paper we obtain the Zagreb indices and coindices of the total graph of the subdivision graph of any graph, generalizing the results of Mohanappriya and Vijayalakshmi [8]. In addition, we compute the Zagreb indices of semi-total point graph and semi-total line graph of the subdivision graph of any graph in terms of the parameters of the parent graph.

The subdivision graph $S(G)$ is the graph obtained from G by inserting a new vertex into each edge of G. The tadpole $T_{n,k}$ is the graph obtained by joining one vertex of the cycle C_n to the one end point of the path P_k. The wheel W_{n+1} is the graph obtained by joining all vertices of C_n to the new vertex.

The Cartesian product $G_1 \times G_2$ of G_1 and G_2 is a graph with vertex set $V(G_1) \times V(G_2)$ in which two vertices (u_1, v_1) and (u_2, v_2) are adjacent if and only if either $u_1 = u_2$ and v_1 is adjacent to v_2 in G_2 or $v_1 = v_2$ and u_1 is adjacent to u_2 in G_1. The Ladder L_n is given by $L_n = K_2 \times P_n$, where P_n is a path on n vertices and K_n is the complete graph on n vertices.

2. Total Graph, Semi-Total Point Graph and Semi-Total Line Graph

The vertices and edges of G are referred to as their elements. The total graph of G, denoted by $T(G)$, is a graph with vertex set $V(T(G)) = V(G) \cup E(G)$ in which two vertices are adjacent if and only if they are adjacent elements or they are incident elements in G [7].

The semi-total point graph of G, denoted by $T_1(G)$, is a graph with vertex set $V(T_1(G)) = V(G) \cup E(G)$ in which two vertices are adjacent if they are adjacent vertices in G or one is vertex and the other is an edge, incident to it [11].

The semi-total line graph of G, denoted by $T_2(G)$, is a graph with vertex set $V(T_2(G)) = V(G) \cup E(G)$ in which two vertices are adjacent if they are adjacent edges in G or one is a vertex of G and the other is an edge, incident to it.

Observation 2.1. If $u \in V(G)$, then $d_{S(G)}(u) = d_G(u)$ and if v is a subdivision vertex, then $d_{S(G)}(v) = 2$.

Observation 2.2. If $u \in V(G)$, then $d_{T(G)}(u) = 2d_G(u)$, $d_{T_1(G)}(u) = 2d_G(u)$ and $d_{T_2(G)}(u) = d_G(u)$.

Observation 2.3. If $e = uv \in E(G)$, then $d_{T(G)}(e) = d_G(u) + d_G(v)$, $d_{T_1(G)}(e) = 2$ and $d_{T_2(G)}(e) = d_G(u) + d_G(v)$.

Without loss of generality, referring to Figure 1, let e and f be adjacent edges at v in G. Let e' and e'' be the subdivision edges of an edge e in $S(G)$ and f' and f'' be the subdivision edges of an edge f in $S(G)$. Let u_e and u_f be the subdivision vertices on edges e and f respectively in $S(G)$.

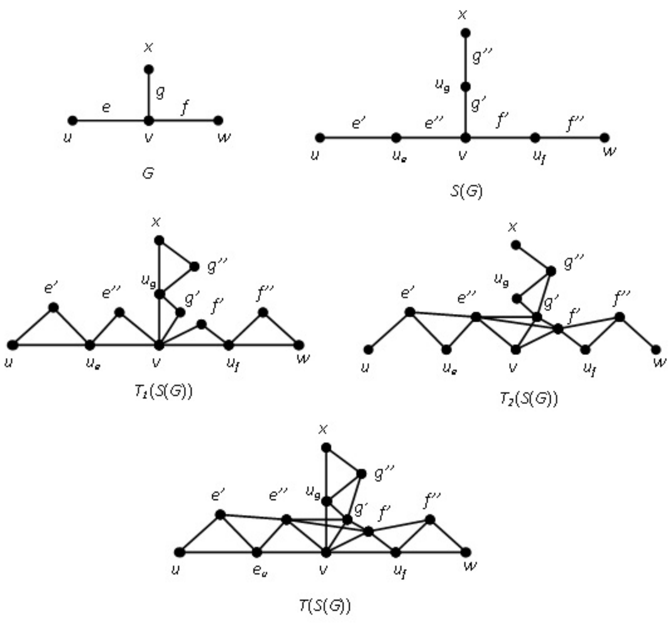

Figure 1: A graph G and its congeners.

The edge set $E(T(S(G)))$ of the total graph of the subdivision graph can be partitioned into sets E_1, E_2, E_3, E_4 and E_5, so that

$$E_1 = \{uu_e\}$$

where $u \in V(G)$ and u_e is the subdivision vertex in $S(G)$,

$$E_2 = \{ue'\}$$

where $u \in V(G)$ and e' is the subdivision edge in $S(G)$,

$$E_3 = \{u_e e'\}$$

where u_e is the subdivision vertex and e' is the subdivision edge in $S(G)$,

$$E_4 = \{e'e''\}$$

where e' and e'' are subdivision edges with common end vertex u_e in $S(G)$ and

$$E_5 = \{e''f'\}$$

where e'' and f' are subdivision edges with common end vertex v in $S(G)$ where $v \in V(G)$.

We can easily check that, $|E_1| = 2m$, $|E_2| = 2m$, $|E_3| = 2m$, $|E_4| = m$, and

$$|E_5| = \sum_{v \in V(G)} \frac{d_G(v)[d_G(v)-1]}{2} = -m + \frac{1}{2} \sum_{v \in V(G)} d_G(v)^2 \,.$$

Observation 2.4. If $u \in V(G)$, then $d_{T(S(G))}(u) = 2d_G(u)$, $d_{T_1(S(G))}(u) = 2d_G(u)$ and $d_{T_2(S(G))}(u) = d_G(u)$.

Observation 2.5. If u_e is a subdivision vertex in $S(G)$, then $d_{T(S(G))}(u_e) = 4$, $d_{T_1(S(G))}(u_e) = 4$ and $d_{T_2(S(G))}(u_e) = 2$.

Observation 2.6. If e' is a subdivision edge with one end vertex $u \in V(G)$, then $d_{T(S(G))}(e') = 2 + d_G(u)$, $d_{T_1(S(G))}(e') = 2$ and $d_{T_2(S(G))}(e') = 2 + d_G(u)$.

3. Zagreb Indices

Theorem 3.1. Let G be a graph with n vertices, m edges and vertex set $V(G)$. Then

$$M_1(T(S(G))) = 24m + 8M_1(G) + \sum_{v \in V(G)} d_G(v)^3 \tag{1}$$

and

$$M_2(T(S(G))) = 16m + 18M_1(G) + M_2(G) + \frac{7}{2} \sum_{v \in V(G)} d_G(v)^3 + \frac{1}{2} \sum_{v \in V(G)} d_G(v)^4 \,. \tag{2}$$

Proof. By referring to Figure 1 and by Observations 2.4–2.6, we have

$$\begin{aligned}
M_1(T(S(G))) &= \sum_{uv\in E(T(S(G)))} \big[d_{T(S(G))}(u) + d_{T(S(G))}(v)\big] \\
&= \sum_{uu_e\in E_1} \big[d_{T(S(G))}(u) + d_{T(S(G))}(u_e)\big] \\
&+ \sum_{ue'\in E_2} \big[d_{T(S(G))}(u) + d_{T(S(G))}(e')\big] \\
&+ \sum_{u_e e'\in E_3} \big[d_{T(S(G))}(u_e) + d_{T(S(G))}(e')\big] \\
&+ \sum_{e'e''\in E_4} \big[d_{T(S(G))}(e') + d_{T(S(G))}(e'')\big] \\
&+ \sum_{e''f'\in E_5} \big[d_{T(S(G))}(e'') + d_{T(S(G))}(f')\big] \\
&= \sum_{uv\in E(G)} \big[2d_G(u) + 4 + 2d_G(v) + 4\big] \\
&+ \sum_{uv\in E(G)} \big[2d_G(u) + 2 + d_G(u) + 2d_G(v) + 2 + d_G(v)\big] \\
&+ \sum_{uv\in E(G)} \big[4 + 2 + d_G(u) + 4 + 2 + d_G(v)\big] \\
&+ \sum_{uv\in E(G)} \big[2 + d_G(u) + 2 + d_G(v)\big] \\
&+ \sum_{e''f'\in E_5} \big[2 + d_G(v) + 2 + d_G(v)\big] \\
&= \big[8m + 2M_1(G)\big] + \big[4m + 3M_1(G)\big] + \big[12m + M_1(G)\big] \\
&+ \big[4m + M_1(G)\big] + \sum_{v\in V(G)} \big[4 + 2d_G(v)\big]\left[\frac{d_G(v)(d_G(v)-1)}{2}\right]
\end{aligned}$$

from which Equation (1) follows.

$$\begin{aligned}
M_2(T(S(G))) &= \sum_{uv\in E(T(S(G)))} d_{T(S(G))}(u)\, d_{T(S(G))}(v) \\
&= \sum_{uu_e\in E_1} d_{T(S(G))}(u)\, d_{T(S(G))}(u_e) \\
&+ \sum_{ue'\in E_2} d_{T(S(G))}(u)\, d_{T(S(G))}(e') \\
&+ \sum_{u_e e'\in E_3} d_{T(S(G))}(u_e)\, d_{T(S(G))}(e') \\
&+ \sum_{e'e''\in E_4} d_{T(S(G))}(e')\, d_{T(S(G))}(e'')
\end{aligned}$$

$$
\begin{aligned}
&+ \sum_{e''f' \in E_5} d_{T(S(G))}(e'')\, d_{T(S(G))}(f') \\
&= \sum_{uv \in E(G)} \big[2d_G(u)(4) + 2d_G(v)(4)\big] \\
&+ \sum_{uv \in E(G)} \big[2d_G(u)(2 + d_G(u)) + 2d_G(v)(2 + d_G(v))\big] \\
&+ \sum_{uv \in E(G)} \big[(4)(2 + d_G(u)) + (4)(2 + d_G(v))\big] \\
&+ \sum_{uv \in E(G)} \big[(2 + d_G(u))(2 + d_G(v))\big] \\
&+ \sum_{e''f' \in E_5} \big[(2 + d_G(v))(2 + d_G(v))\big] \\
&= \big[8M_1(G)\big] + \left[4M_1(G) + 2 \sum_{uv \in E(G)} \big[d_G(u)^2 + d_G(v)^2\big]\right] \\
&+ \big[16m + 4M_1(G)\big] + [4m + 2M_1(G) + M_2(G)] \\
&+ \sum_{v \in V(G)} \big[2 + d_G(v)\big]^2 \left[\frac{d_G(v)(d_G(v) - 1)}{2}\right]
\end{aligned}
$$

which directly implies Equation (2). □

Corollary 3.2. [8] Let $T_{n,k}$ be the tadpole graph, W_{n+1} be the wheel and L_n be the ladder. Then

$$
\begin{aligned}
M_1(T(S(T_{n,k}))) &= 64(n + k) + 28, \\
M_1(T(S(W_{n+1}))) &= 147n + 8n^2 + n^3, \\
M_1(T(S(L_n))) &= 270n - 284\,.
\end{aligned}
$$

Theorem 3.3. Let G be a graph with n vertices and m edges. Then

$$M_1(T_1(S(G))) = 24m + 4M_1(G) \tag{3}$$

and

$$M_2(T_1(S(G))) = 16m + 12M_1(G)\,. \tag{4}$$

Proof. By referring to Figure 1 we see that the edge set $E(T_1(S(G))) = E_1 \cup E_2 \cup$

E_3. Therefore by Observations 2.4–2.6, we have

$$\begin{aligned}
M_1(T_1(S(G))) &= \sum_{uv\in E(T_1(S(G)))} \big[d_{T_1(S(G))}(u) + d_{T_1(S(G))}(v)\big]\\
&= \sum_{uu_e\in E_1} \big[d_{T_1(S(G))}(u) + d_{T_1(S(G))}(u_e)\big]\\
&+ \sum_{ue'\in E_2} \big[d_{T_1(S(G))}(u) + d_{T_1(S(G))}(e')\big]\\
&+ \sum_{u_e e'\in E_3} \big[d_{T_1(S(G))}(u_e) + d_{T_1(S(G))}(e')\big]\\
&= \sum_{uv\in E(G)} \big[2d_G(u) + 4 + 2d_G(v) + 4\big]\\
&+ \sum_{uv\in E(G)} \big[2d_G(u) + 2 + 2d_G(v) + 2\big]\\
&+ \sum_{uv\in E(G)} \big[4 + 2 + 4 + 2\big] = \big[8m + 2M_1(G)\big]\\
&+ \big[4m + 2M_1(G)\big] + 12m
\end{aligned}$$

from which Equation (3) immediately follows.

$$\begin{aligned}
M_2(T_1(S(G))) &= \sum_{uv\in E(T_1(S(G)))} d_{T_1(S(G))}(u)\, d_{T_1(S(G))}(v)\\
&= \sum_{uu_e\in E_1} d_{T_1(S(G))}(u)\, d_{T_1(S(G))}(u_e)\\
&+ \sum_{ue'\in E_2} d_{T_1(S(G))}(u)\, d_{T_1(S(G))}(e')\\
&+ \sum_{u_e e'\in E_3} d_{T_1(S(G))}(u_e)\, d_{T_1(S(G))}(e')\\
&= \sum_{uv\in E(G)} \big[2d_G(u)(4) + 2d_G(v)(4)\big]\\
&+ \sum_{uv\in E(G)} \big[2d_G(u)(2) + 2d_G(v)(2)\big]\\
&+ \sum_{uv\in E(G)} \big[(4)(2) + (4)(2)\big]
\end{aligned}$$

resulting in Equation (4). □

Theorem 3.4. Let G be a graph with n vertices, m edges and vertex set $V(G)$. Then

$$M_1(T_2(S(G))) = 12m + 5M_1(G) + \sum_{v \in V(G)} d_G(v)^3 \quad (5)$$

and

$$M_2(T_2(S(G))) = 8m + 6M_1(G) + M_2(G) + \frac{5}{2} \sum_{v \in V(G)} d_G(v)^3 + \frac{1}{2} \sum_{v \in V(G)} d_G(v)^4. \quad (6)$$

Proof. For the edge set of $T_2(S(G))$ it holds $E(T_2(S(G))) = E_2 \cup E_3 \cup E_4 \cup E_5$. Therefore by Observations 2.4–2.6,

$$\begin{aligned}
M_1(T_2(S(G))) &= \sum_{uv \in E(T_2(S(G)))} \left[d_{T_2(S(G))}(u) + d_{T_2(S(G))}(v)\right] \\
&= \sum_{ue' \in E_2} \left[d_{T_2(S(G))}(u) + d_{T_2(S(G))}(e')\right] \\
&+ \sum_{u_e e' \in E_3} \left[d_{T_2(S(G))}(u_e) + d_{T_2(S(G))}(e')\right] \\
&+ \sum_{e'e'' \in E_4} \left[d_{T_2(S(G))}(e') + d_{T_2(S(G))}(e'')\right] \\
&+ \sum_{e''f' \in E_5} \left[d_{T_2(S(G))}(e'') + d_{T_2(S(G))}(f')\right] \\
&= \sum_{uv \in E(G)} \left[d_G(u) + 2 + d_G(u) + d_G(v) + 2 + d_G(v)\right] \\
&+ \sum_{uv \in E(G)} \left[2 + 2 + d_G(u) + 2 + 2 + d_G(v)\right] \\
&+ \sum_{uv \in E(G)} \left[2 + d_G(u) + 2 + d_G(v)\right] \\
&+ \sum_{e''f' \in E_5} \left[2 + d_G(v) + 2 + d_G(v)\right] \\
&= \left[4m + 2M_1(G)\right] + \left[8m + M_1(G)\right] + \left[4m + M_1(G)\right] \\
&+ \sum_{v \in V(G)} (4 + 2d_G(v)) \left[\frac{d_G(v)(d_G(v) - 1)}{2}\right]
\end{aligned}$$

implying Equation (5).

$$\begin{aligned}
M_2(T_2(S(G))) &= \sum_{uv\in T_2(S(G))} d_{T_2(S(G))}(u)\, d_{T_2(S(G))}(v) \\
&= \sum_{ue'\in E_2} d_{T_2(S(G))}(u)\, d_{T_2(S(G))}(e') \\
&+ \sum_{u_e e'\in E_3} d_{T_2(S(G))}(u_e)\, d_{T_2(S(G))}(e') \\
&+ \sum_{e'e''\in E_4} d_{T_2(S(G))}(e')\, d_{T_2(S(G))}(e'') \\
&+ \sum_{e''f'\in E_5} d_{T_2(S(G))}(e'')\, d_{T_2(S(G))}(f') \\
&= \sum_{uv\in E(G)} \big[d_G(u)\,(2+d_G(u)) + d_G(v)\,(2+d_G(v))\,\big] \\
&+ \sum_{uv\in E(G)} \big[2\,(2+d_G(u)) + 2\,(2+d_G(v))\,\big] \\
&+ \sum_{uv\in E(G)} (2+d_G(u))(2+d_G(v)) \\
&+ \sum_{e''f'\in E_5} (2+d_G(v))(2+d_G(v)) \\
&= \sum_{uv\in E(G)} \big[2\,(d_G(u)+d_G(v)) + \big(d_G(u)^2+d_G(v)^2\big)\,\big] \\
&+ \sum_{uv\in E(G)} \big[8 + 2\,(d_G(u)+d_G(v))\,\big] \\
&+ \sum_{uv\in E(G)} \big[4 + 2\,(d_G(u)+d_G(v)) + d_G(u)\,d_G(v)\big] \\
&+ \sum_{v\in V(G)} \big[4 + 4d_G(v) + d_G(v)^2\big] \left[\frac{d_G(v)(d_G(v)-1)}{2}\right] \\
&= \big[2M_1(G) + \sum_{v\in V(G)} (d_G(v))^3\big] + \big[8m + 2M_1(G)\big] \\
&+ \big[4m + 2M_1(G) + M_2(G)\big] + \big[2M_1(G) - 4m \\
&+ 2\sum_{v\in V(G)} d_G(v)^3 - 2M_(G) + \frac{1}{2}\sum_{v\in V(G)} d_G(v)^4 - \frac{1}{2}\sum_{v\in V(G)} d_G(v)^3\big]
\end{aligned}$$

which after rearrangements yields Equation (6). □

4. Zagreb Coindices

In [6] the following results were established:

Let G be a graph with n vertices and m edges. Then

$$\begin{aligned}\overline{M_1}(G) &= 2m(n-1) - M_1(G),\\ \overline{M_2}(G) &= 2m^2 - \frac{1}{2}M_1(G) - M_2(G)\,.\end{aligned}$$

In addition,
$T(S(G))$ has $n+3m$ vertices and $6m + \frac{1}{2}M_1(G)$ edges.
$T_1(S(G))$ has $n+3m$ vertices and $6m$ edges.
$T_2(S(G))$ has $n+3m$ vertices and $4m + \frac{1}{2}M_1(G)$ edges.

Taking the above in mind, along with the results of Section 3, we get:

Theorem 4.1. Let G be a graph with n vertices, m edges and vertex set $V(G)$. Then

$$\begin{aligned}\overline{M_1}(T(S(G))) &= 12m(n+3m-3) + (n+3m-9)M_1(G) - \sum_{v\in V(G)} d_G(v)^3,\\ \overline{M_2}(T(S(G))) &= m(72m-28) + \left[12m - 22 + \frac{1}{2}M_1(G)\right] M_1(G) - M_2(G)\\ &- 4\sum_{v\in V(G)} d_G(v)^3 - \frac{1}{2}\sum_{v\in V(G)} d_G(v)^4\,,\\ \overline{M_1}(T_1(S(G))) &= 12m(n+3m-3) - 4M_1(G),\\ \overline{M_2}(T_1(S(G))) &= 72m^2 - 28m - 14M_1(G),\\ \overline{M_1}(T_2(S(G))) &= 4m(2n+6m-5) + (n+3m-6)M_1(G) - \sum_{v\in V(G)} d_G(v)^3,\\ \overline{M_2}(T_2(S(G))) &= m(32m-14) + \left[8m + \frac{1}{2}M_1(G) - \frac{17}{2}\right] M_1(G) - M_2(G),\\ &- 3\sum_{v\in V(G)} d_G(v)^3 - \frac{1}{2}\sum_{v\in V(G)} d_G(v)^4\,.\end{aligned}$$

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References

[1] A. R. Ashrafi, T. Došlić and A. Hamzeh, The Zagreb coindices of graph operations, *Discrete Appl. Math.* **158** (2010) 1571–1578.

[2] B. Borovićanin, K. C. Das, B. Furtula and I. Gutman, Bounds for Zagreb indices, *MATCH Commun. Math. Comput. Chem.* **78** (2017) 17–100.

[3] J. Devillers and A. T. Balaban, *Topological Indices and Related Descriptors in QSAR and QSPR*, Gordon & Breach Science, Amsterdam, 1999.

[4] T. Došlić, Vertex–weighted Wiener polynomials for composite graphs, *Ars Math. Comtemp.* **1** (2008) 66–80.

[5] I. Gutman, Degree–based topological indices, *Croat. Chem. Acta* **86** (2013) 351–361.

[6] I. Gutman, B. Furtula, Ž. Kovijanić Vukićević and G. Popivoda, On Zagreb indices and coindices, *MATCH Commun. Math. Comput. Chem.* **74** (2015) 5–16.

[7] F. Harary, *Graph Theory*, Addison Wesley, Reading, 1969.

[8] G. Mohanappriya and D. Vijayalakshmi, Topological indices of total graph of subdivision graphs, *Ann. Pure Appl. Math.* **14** (2017) 231–235.

[9] H. S. Ramane, V. V. Manjalapur and I. Gutman, General sum-connectivity index, general product-connectivity index, general Zagreb index and coindices of line graph of subdivision graphs, *AKCE Int. J. Graphs Comb.* **14** (2017) 92–100.

[10] P. S. Ranjini, V. Lokesha and I. N. Cangül, On the Zagreb indices of the line graphs of the subdivision graphs, *Appl. Math. Comput.* **218** (2011) 699–702.

[11] E. Sampathkumar and S. B. Chikkodimath, Semi-total graphs of a graph, I, II, III, *J. Karnatak Univ. Sci.* **18** (1973) 274–280.

[12] R. Todeschini and V. Consonni, *Molecular Descriptors for Chemoinformatics*, Wiley–VCH Verlag GmbH & Co. KGaA, Weinheim, 2009.

Harishchandra S. Ramane
Department of Mathematics,
Karnatak University,
Dharwad, India
E-mail: hsramane@yahoo.com

Saroja Y. Talwar
Department of Mathematics,

Karnatak University,
Dharwad, India
E-mail: sarojaytalwar@gmail.com

Ivan Gutman
Faculty of Science,
University of Kragujevac,
Kragujevac, Serbia
E-mail: gutman@kg.ac.rs