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## Modeling and solving a three-stage fixed charge transportation problem considering stochastic demand and price

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### Abstract

This paper considers a three-stage fixed charge transportation problem regarding stochastic demand and price. The objective of the problem is to maximize the profit for supplying demands. Three kinds of costs are presented here: variable costs that are related to amount of transportation cost between a source and a destination. Fixed charge exists whenever there is a transfer from a source to a destination, and finally, shortage cost that incurs when the manufacturer does not have enough products for supplying customer's demand. The model is formulated as a mixed integer programming problem and is solved using a multi-criteria scenario based solution approach to find the optimal solution. Mean, standard deviation, and coefficient of variation are compared as the acceptable criteria to decide about the best solution.

**Keywords:** Fixed charge transportation problem; stochastic optimization; multi-criteria scenario-based solution; mean; standard deviation; coefficient of variation.

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### 1. Introduction

A supply chain is a network of suppliers, manufacturers, transporters, distribution centres, retailers and customers (N Jawahar & Balaji, 2009). The problem of transportation from companies to customers through the warehouses and distribution centres is an important problem in supply chain. The cost of distribution accounts for about 30 percent of the product's cost price (Diaby, 1991; Eskigun et al., 2005). Therefore distribution problem is an important problem for both of the manufacturer and customer. The classical transportation problem (TP) refers to a special class of linear programming. It is well-known as a basic network problem. The first formulation and discussion of the classical transportation problem as a network optimization problem was introduced by Hitchcock (Hitchcock, 1941). The objective is to find a way to

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transport homogeneous products from several sources to several destinations so that the total cost can be minimized. In a transportation problem, when fixed cost is also taken into account, the problem is known as fixed charge transportation problem (FCTP). The objective of an FCTP is to find the combination of routes that minimises the total distribution costs satisfying the supply and demand constraints (Vinay & Sridharan, 2012). So, the fixed-charge transportation problem is an extension of the classical transportation problem that considers two kinds of cost (variable and fixed costs) (Raj & Rajendran, 2011; Schaffer & O'Leary, 1989). Variable cost depends on per unit of transported and linearly increases with it. Fixed charge incurs whenever a nonzero quantity is transported from a source to a destination (Adlakha & Kowalski, 1999; N. Jawahar & Balaji, 2009). In FCTP, the parameters (for example variable costs, fixed charges, price and demand) can be deterministic and Non-deterministic. Some research can be refer as deterministic (Adlakha, Kowalski, & Lev, 2010; N. Jawahar & Balaji, 2009; N. Jawahar & Balaji, 2012; Lotfi & Tavakkoli-Moghaddam, 2013), etc. On the other hand, few works are undertaken with non-deterministic parameters in FCTP. Non-deterministic parameters can be different approaches such as fuzzy (Kundu, Kar, & Maiti, 2014; Yang & Liu, 2007), interval (Safi & Razmjoo, 2013), chaos, stochastic etc. To the best of our knowledge, there are not any works about FCTP when parameters of demand and price are stochastic in a 3-stage supply chain. In this paper, we focus on a 3-stage FCTP. We try to find quantity of transported products from a manufacturer plant to a distribution centre and from a distribution centre to a retailer and a retailer to a customer when the parameters of demand and price are nondeterministic to obtain maximum income.

The organization of this paper is as follows: Section 2 presents the literature review of FCTP to find gaps. Section 3 describes the mathematical model and its descriptions. Section 4 explains the solution methodology. Sensitivity analysis is presented in Section 5. Finally, in Section 6, conclusions are provided and some areas of further research are then stated.

## **2. Literature review**

Literature review includes two sections: deterministic parameters and nondeterministic parameters in FCTP. Further, number of stages is considered in these two sections. There are many studies regarding deterministic parameters for FCTP. Review is categorised based on number of stage.

Adlakha et al. (Adlakha et al., 2010) proposes a branching method for the solution of the single stage fixed charge transportation problem. Adlakha and Kowalski (Adlakha & Kowalski, 2010) develop a heuristic algorithm for its solution for the same problem. The algorithm is based upon the Balinski (Balinski, 1961) approximation solution method for a fixed cost transportation problem. This method is useful in dealing with the fundamental nonlinear problems. Kim et al. (Kim et al., 2011) consider the fixed-charge capacitated network design problem with turn penalties. They present a mixed integer programming model for the problem and suggested a two-phase heuristic algorithm to solve the problem. The objective of the problem is to minimize the sum of flow costs for routing the commodities demand, fixed costs for using arcs and penalty costs for flows with 90-degree turns. Jawahar and Balaji (N. Jawahar & Balaji, 2012) propose a genetic algorithm (GA) based heuristic to the multi-period fixed charge distribution problem associated with backorder and inventories. The objective is to determine the size of the shipments, backorder and inventories at each period, so that, the total cost incurred during the entire period towards transportation, backorder and inventories is minimum. The model is formulated as pure integer nonlinear programming and 0–1 mixed integer linear programming problems, and proposes a GA based heuristic to provide solution to the above problem. Adlakha et al. (Adlakha, Kowalski, Wang, Lev, & Shen, 2014) propose a new approach of approximating and solving a single stage FCTP by proposing novel approximations for the objective function of the FCTP to obtain lower bounds for the optimal solution. Lev and Kowalski (Lev & Kowalski,

2011) formulate single stage fixed-charge problems with polynomials. Using polynomial formulations. They show structural similarity between different kinds of linear and fixed charge formulations. Lotfi and Tavakkoli-Moghaddam (Lotfi & Tavakkoli-Moghaddam, 2013) propose a genetic algorithm using priority-based encoding (pb-GA) for linear and nonlinear fixed charge transportation problems in which new operators for more exploration are proposed. They modify a priority-based decoding procedure proposed by Gen et al. (Gen, Kumar, & Ryul Kim, 2005) to adapt with the FCTP structure. Pintea et al. (Pintea, Sitar, Hajdu-Macelaru, & Petrica, 2012) describe some hybrid techniques for solving the fixed charged transportation problem. The problem is a two chain supply network. Classical nearest neighbor algorithm is used basically to find the best distribution centres.

Now we point some studies about FCTP with Non-deterministic parameters. A few works are done about uncertain FCTP. Safi and Razmjoo (Safi & Razmjoo, 2013) consider the fixed charge transportation problem under uncertainty, particularly when parameters are given in interval forms. All of parameters (i.e., variable costs, fixed charges, supply and demand parameters) are in interval forms. In this case it is assumed that both cost and constraint parameters are arrived in interval numbers. Considering two different order relations for interval numbers, two solution procedures are developed in order to obtain an optimal solution for interval fixed charge transportation problem (IFCTP). Kundu et al (Kundu et al., 2014) consider two fixed charge transportation problems with type-2 fuzzy parameters. Unit transportation costs, fixed costs in the first problem and unit transportation costs, fixed costs, supplies and demands in the second problem are type-2 fuzzy variables.

A defuzzification method of general type-2 fuzzy variable is outlined and compared numerically with geometric defuzzification method. Yang and Liu (Yang & Liu, 2007) investigate the fixed charge solid transportation problems under a fuzzy environment, in which the direct costs, the fixed charges, the supplies, the demand, and the conveyance capacities have been considered as fuzzy variables. They designed a hybrid intelligent algorithm based on the fuzzy simulation technique and tabu search algorithm to solve them. Table presents a review on some papers. The first row shows characteristics of this study. According to the above discussion, some gaps are considered in this paper:

- Most of the works in the literature consider the FCTP with deterministic parameters. Few works consider nondeterministic parameters, as seen in Table 1.
- Demand and transportation costs are considered under uncertainty (see Table 1). We consider price and demand in an uncertain environment.
- In nondeterministic works, fuzzy and interval are two main nondeterministic approaches. To the best of our knowledge we cannot find any papers with stochastic approach (about demand and price) in 3-stage supply chain.

Finally, based on the above analyses of literature review in Table 1, this paper is proposed a 3-stage FCTP with stochastic demand and price and it is solved by a scenario-based solution methodology.

**Table 1: A review of planning under uncertainty for FCTP**

<b>Paper</b>	<b>solution method</b>	<b>price</b>	<b>demand</b>	<b>stage</b>
Hajiaghaei et al. 2010	spanning tree-based genetic algorithm	–	deterministic	1
Xie and Jia 2010	spanning tree-based genetic algorithm	–	deterministic	1
Vinay Panicker et al. 2012	genetic algorithm (GA) based heuristic	–	deterministic	3
Adlakha et al. 2007	Heuristic algorithm	–	deterministic	1
Molla-Alizadeh et al. 2013	artificial immune algorithm (AIA) and GA based on the spanning tree and Prüfer number representation	–	deterministic	2
Xie and Jia 2012	the minimum cost flow-based genetic algorithm named NFCTP-HGA	–	deterministic	1
Kim et al. 2011	two-phase heuristic algorithm	–	deterministic	1
Yang and Liu 2007	hybrid intelligent algorithm based on the fuzzy simulation technique and tabu search algorithm	–	Fuzzy	
El-Sherbiny and Alhamali 2013	Hybrid Particle Swarm algorithm with artificial Immune Learning (HPSIL)	–	deterministic	1
Raj and Rajendran 2011	hybrid genetic algorithm	–	deterministic	1
El-Sherbiny 2012	alternate Mutation based Artificial Immune (MAI) algorithm	–	deterministic	1
Safi and Razmjoo 2013	Equivalent crisp problem using order relation	–	Interval	1
Kundu et al. 2014	A defuzzification method of general type-2 fuzzy variable, geometric defuzzification method	–	Fuzzy	1
Raj & Rajendran 2012	GA	–	deterministic	2
<b>This study</b>	<b>multi-criteria scenario-based solution approach</b>	<b>stochastic</b>	<b>stochastic</b>	<b>3</b>

### 3. Problem description

The presented three-stage fixed charge transportation problem includes  $n$  plants,  $m$  distributors,  $l$  retailers and  $d$  customers (see Figure 1). The characteristics of the model are as follows:

- The model is scenario-based. Demand and price are nondeterministic and it could be different for each scenario. Scenarios are created randomly in the three groups with poor, medium and high logic.
- The number and capacity of all facilities, and all cost parameters are predetermined.
- Each of the  $l$  retailers can ship products to any of the  $d$  customers. In other words, a customer can be supplied with products from more than a retailer.

- Each of the  $m$  distribution centres can ship products to any of the  $l$  retailers. In other words, a retailer can replenish the inventory from multiple distribution centres.
- Each of the  $n$  plants can ship products to any of the  $m$  distribution centres. In other words, a distribution centre can replenish the inventory from multiple plants.
- The production shortage is allowed. The backorder penalty cost is considered.

Notations are presented as follows:

$i$ : Set of plants ( $i = 1$  to  $n$ )

$j$ : Set of distribution centres ( $j = 1$  to  $m$ )

$r$ : Set of retailers ( $r = 1$  to  $l$ )

$K$ : Set of customers ( $k = 1$  to  $d$ )

$S$ : Demands and price scenarios ( $s = 1$  to  $g$ )

$x_{ijs}$ : Number of quantity transported from plant  $i$  to distributor  $j$  under scenario  $s$ .

$c_{ij}$ : Unit cost of transportation between plant  $i$  and distributor  $j$ .

$f_{ij}$ : Fixed transportation cost between plant  $i$  and distributor  $j$ .

$u_{jrs}$ : Number of quantity transported from distributor  $j$  to retailer  $r$  under scenario  $s$ .

$b_{jr}$ : Unit cost of transportation between distributor  $j$  and retailer  $r$ .

$o_{jr}$ : Fixed transportation cost between distributor  $j$  and retailer  $r$ .

$t_{rks}$ : Number of quantity transported from retailer  $r$  to customer  $k$  under scenario  $s$ .

$v_{rk}$ : Unit cost of transportation between retailer  $r$  and customer  $k$ .

$q_{rk}$ : Fixed transportation cost between retailer  $r$  and customer  $k$ .

$D_{ks}$ : Demand at customer  $k$  under scenario  $s$ .

$P_{ks}$ : Sales price at customer  $k$  under scenario  $s$ .

$H_{ks}$ : Number of units backordered at customer  $k$  under scenario  $s$ .

$Hc_k$ : Unit cost of backorder at customer  $k$ .

$Am_i$ : Capacity at plant  $i$ .

$Ad_j$ : Capacity at distributor  $j$ .

$Ar_r$ : Capacity at retailer  $r$ .

$z_{ijs}$ : Binary variable that specifies whether the product is distributed from plant  $i$  to distribution centre  $j$  under scenario  $s$ . ( $z_{ijs} = 0$  or  $1$ )

$y_{jrs}$ : Binary variable that specifies whether the product is distributed from distribution center  $j$  to retailer  $r$  under scenario  $s$ . ( $y_{jrs} = 0$  or  $1$ )

$w_{rks}$ : Binary variable that specifies whether the product is distributed from retailer  $r$  to customer  $k$  under scenario  $s$ . ( $w_{rks} = 0$  or  $1$ )

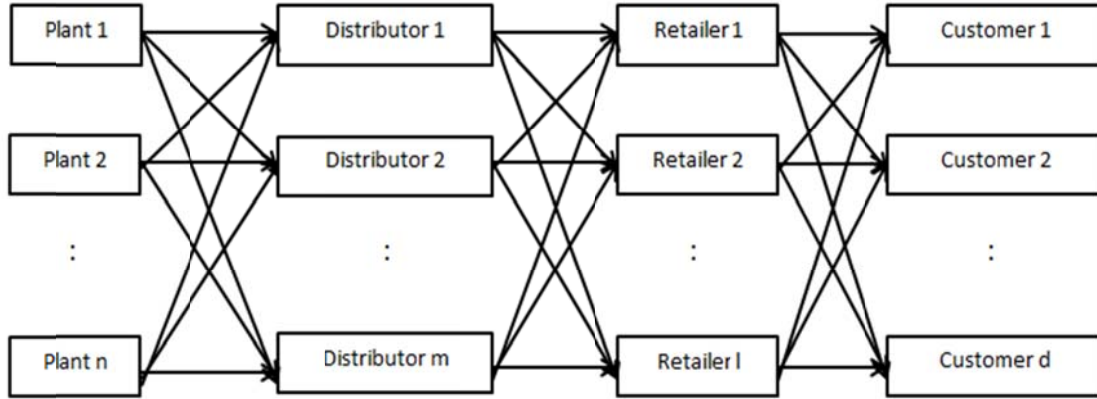


Figure 1: Proposed three-stage FCTP.

The fixed-charge transportation problem is formulated as follows:

Maximize:

$$Z = \sum_{s=1}^g \sum_{k=1}^d P_{ks} D_{ks} - \sum_{s=1}^g \sum_{i=1}^n \sum_{j=1}^m (c_{ij} x_{ijs} + f_{ij} z_{ijs}) - \sum_{s=1}^g \sum_{j=1}^m \sum_{r=1}^l (b_{jr} u_{jrs} + o_{jr} y_{jrs}) - \sum_{s=1}^g \sum_{r=1}^l \sum_{k=1}^d (v_{rk} t_{rks} + q_{rk} w_{rks}) - \sum_{s=1}^g \sum_{k=1}^d H_{ks} H c_k \tag{1}$$

Subject to:

$$\sum_{j=1}^m x_{ijs} \leq A m_i \quad i=1, \dots, n \quad s=1, \dots, g \tag{2}$$

$$\sum_{i=1}^n x_{ijs} \leq A d_j \quad j=1, \dots, m \quad s=1, \dots, g \tag{3}$$

$$\sum_{j=1}^m u_{jrs} \leq A r_r \quad r=1, \dots, l \quad s=1, \dots, g \tag{4}$$

$$\sum_{r=1}^l t_{rks} \geq 0.8 D_{ks} \quad k=1, \dots, d \quad s=1, \dots, g \tag{5}$$

$$\sum_{i=1}^n x_{ijs} = \sum_{r=1}^l u_{jrs} \quad j=1, \dots, m \quad s=1, \dots, g \tag{6}$$

$$\sum_{j=1}^m u_{jrs} = \sum_{k=1}^d t_{rks} \quad r=1, \dots, l \quad s=1, \dots, g \tag{7}$$

$$x_{ijs}, u_{jrs}, t_{rks} \geq 0 \quad (\text{for all } i, j, r, k \text{ and } s) \tag{8}$$

$$z_{ijs} \leq x_{ijs} \leq z_{ijs} M \quad i=1, \dots, n \quad j=1, \dots, m \quad s=1, \dots, g \tag{9}$$

$$y_{jrs} \leq u_{jrs} \leq y_{jrs} M \quad j=1, \dots, m \quad r=1, \dots, l \quad s=1, \dots, g \tag{10}$$

$$w_{rks} \leq t_{rks} \leq w_{rks} M \quad r=1, \dots, l \quad k=1, \dots, d \quad s=1, \dots, g \tag{11}$$

$$z_{ijs} \in \{0, 1\} \quad i=1, \dots, n \quad j=1, \dots, m \quad s=1, \dots, g \tag{12}$$

$$y_{jrs} \in \{0, 1\} \quad j=1, \dots, m \quad r=1, \dots, l \quad s=1, \dots, g \tag{13}$$

$$w_{rks} \in \{0, 1\} \quad r=1, \dots, l \quad k=1, \dots, d \quad s=1, \dots, g \tag{14}$$

The objective function (1) is to maximize the total profits that is calculated by total sales minus total costs. The total cost is the total cost transportation incurred in supplying the products from

plants to customers through distributors and retailers, considering the possible combination of routes, plus shortage costs. Constraint (2) represents plant capacity constraint. This constraint maintains that the product quantity which is distributed from the plant to the distribution centers must be less than or equal to capacity of the plant. Constraint (3) denotes distributor capacity constraint. This constraint implies that the quantity of products received in a distribution center from plants must be less than or equal to the capacity of the distribution center. Constraint (4) indicates retailer capacity constraint. This constraint maintains that the quantity of products received in a retailer from distribution centers must be equal to or less than the capacity of the retailer. Constraint (5) denotes customer demand constraint. This constraint maintains that the retailers must provide at least 80 percent of customer's demand. So, the shortage is allowed. Constraint (6) is the balance constraints of distributors. This maintains that all entering flows to a distribution center and all issuing flows from it are equal. Constraint (7) is the balance constraints of retailers. This constraint guarantees that all entering flows to a retailer and all issuing flows from it are equal. Constraint (8) ensures the non-negativity nature of decision variables. Constraint (9) to (14) asserts the 0–1 binary nature of the binary variable. These constraints maintain that if  $x_{ijs} > 0$ , then  $z_{ijs}=1$ , if  $u_{jrs} > 0$ , then  $y_{jrs} = 1$  and if  $t_{rks} > 0$ , then  $w_{rks}=1$ .

#### 4. Solution methodology

Scenario-based approaches for solving the stochastic programming problems are efficient methodologies (Kaut & Wallace, 2003; Listes, 2007). In this paper, the problem is solved with using a multi criteria scenario based solution approach, that the first time is presented by Soleimani (Soleimani, Seyyed-Esfahani, & Shirazi, 2013). Mean, standard deviation and coefficient of variation, which are the mentioned criteria for finding the optimal solution.

The solution of this mathematical model consists of two plants, two distribution centers, two retailers and two customers. It is undertaken for two products and 16 scenarios. Then, 2 various possibilities for demands and prices based on the 4 range of the data in Table 2 are created randomly. A set of system's parameters are presented in Table 3.

Table 2: Range of the demand and price in scenarios.

	Low-quality	Medium-quality	High-quality	Very High-quality
<b>Demand</b>	100–200	200–300	300–400	400–500
<b>Price</b>	10000–12000	12000–14000	14000–16000	16000–18000

Table 3: Parameters of the computational study

Parameter	value
unit cost of transportation between plant $i$ and distributor $j$ ( $c_{ij}$ )	$c_{11} = 100$ $c_{12} = 200$ $c_{21} = 150$ $c_{22} = 200$
fixed transportation cost between plant $i$ and distributor $j$ ( $f_{ij}$ )	$f_{11} = 500$ $f_{12} = 300$ $f_{21} = 300$ $f_{22} = 400$
unit cost of transportation between distributor $j$ and retailer $r$ ( $b_{jr}$ )	$b_{11} = 500$

	$b_{12} = 300$
	$b_{21} = 100$
	$b_{22} = 200$
fixed transportation cost between distributor $j$ and retailer $r$ ( $o_{jr}$ )	$o_{11} = 550$
	$o_{12} = 400$
	$o_{21} = 200$
	$o_{22} = 350$
unit cost of transportation between retailer $r$ and customer $k$ ( $v_{rk}$ )	$v_{11} = 300$
	$v_{12} = 150$
	$v_{21} = 200$
	$v_{22} = 200$
fixed transportation cost between retailer $r$ and customer $k$ ( $q_{rk}$ )	$q_{11} = 500$
	$q_{12} = 250$
	$q_{21} = 350$
	$q_{22} = 250$
unit cost of backorder at customer $k$ ( $Hc_k$ )	$Hc_1 = 10000$
	$Hc_2 = 15000$
capacity at plant $I$ ( $Am_i$ )	500
capacity at distributor $j$ ( $Ad_j$ )	500
capacity at retailer $r$ ( $Ar_r$ )	500

The solution steps are illustrated as follows:

Step 1: all scenarios are solved by LINGO and the optimum points are obtained and recorded as candidate solutions for final optimal point. The results are illustrated in Table 4. Figure 2 presents the objective function values of 16 scenarios

**Table 4: objective function values of 16 scenarios.**

Scenarios	S1	S2	S3	S4	S5	S6	S7	S8
Profit	251675	248507.5	391845	962130	91978.75	349452.5	910430	2.67E+05
Scenarios	S9	S10	S11	S12	S13	S14	S15	S16
Profit	407075	365657.5	5.07E+05	3.11E+05	258617.5	850635	2.82E+05	1.46E+06



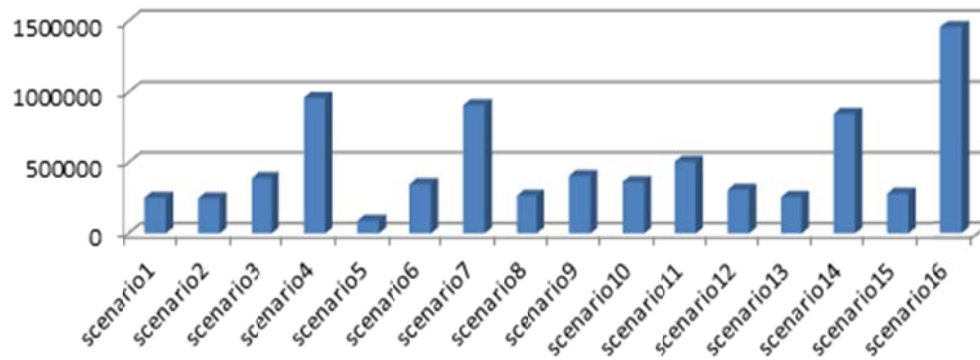


Figure 2: Objective function values of 16 scenarios

- Step 2: scenarios in the three groups with poor, medium and high logic are classified. 4 scenarios with poor logic have  $0.5w$  weight. 6 scenarios with medium logic have  $w$  weight. 6 scenarios with high logic have  $2w$  weight. The probability of scenarios with poor logic is 0.025. The probability of scenarios with medium logic is 0.05. The probability of scenarios with high logic is 0.1. After obtaining the answer of 16 scenarios, weighted average of answers are calculated with using the probability of scenarios.
- Step 3: the initial response of 16 scenarios (candidate solutions) are evaluated in all scenarios and objective function values are recorded. We have 16 solutions that should be evaluated in 16 different scenarios so the model should be solved 256 times. In each row, the performances of a candidate solution are evaluated in 16 scenarios (objective function values). Then, weighted average of answers is calculated according to the probability of scenarios. The results are presented in Table 5.
- Step 4: We used three criteria (mean, standard deviation, and coefficient of variation) as acceptable criteria to select the best solution. The last three columns of Table 5 are the criteria of making final decision of the best solution in various situations (see Figures 3, 4 and 5).
- Step 5: optimal solution is selected based on the analyses of three criteria and appropriate sensitivity analyses are undertaken to determine the reliability of the developed solution approach.

Table 5: Scenario-based solution approach for stochastic model

	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	MEAN	SD	CV
Solution 1	251675	120857.5	116195	224830	62228.75	119477.5	232030	56387.5	250475	120257.5	116615	56207.5	124217.5	240035	58187.5	225550	2375226	75638.83	0.031845
Solution 2	506975	248507.5	243845	480130	126053.8	247127.5	487330	120212.5	505775	247907.5	244265	120032.5	251867.5	495335	122012.5	480850	4928226	156192.6	0.031693
Solution 3	802975	396507.5	391845	776130	200053.8	395127.5	783330	194212.5	801775	395907.5	392265	194032.5	399867.5	791335	196012.5	776850	7888226	249733.2	0.031659
Solution 4	988975	489507.5	484845	962130	246553.8	488127.5	969330	240712.5	987775	488907.5	485265	240532.5	492867.5	977335	242512.5	962850	9748226	308531.6	0.031665
Solution 5	370675	180357.5	175695	343830	91978.75	178977.5	351030	86137.5	369475	179757.5	176115	85957.5	183717.5	359035	87937.5	344550	3565226	113154.7	0.031738
Solution 6	711625	350832.5	346170	684780	177216.3	349452.5	691980	171375	710425	350232.5	346590	171195	354192.5	699985	173175	685500	6974726	220859.5	0.031666
Solution 7	930075	460057.5	455395	903230	231828.8	458677.5	910430	225987.5	928875	459457.5	455815	225807.5	463417.5	918435	227787.5	903950	9159226	289911.2	0.031652
Solution 8	1095175	542607.5	537945	1068330	273103.8	541227.5	1075530	267262.5	1093975	542007.5	538365	267082.5	545967.5	1083535	269062.5	1069050	10810226	342106.7	0.031647
Solution 9	408275	199157.5	194495	381430	101378.8	197777.5	388630	95537.5	407075	198557.5	194915	95357.5	202517.5	396635	97337.5	382150	3941226	125022.8	0.031722
Solution 10	742475	366257.5	361595	715630	184928.8	364877.5	722830	179087.5	741275	365657.5	362015	178907.5	369617.5	730835	180887.5	716350	7283226	230610.1	0.031663
Solution 11	1032275	511157.5	506495	1005430	257378.8	509777.5	1012630	251537.5	1031075	510557.5	506915	251357.5	514517.5	1020635	253337.5	1006150	10181226	322220.6	0.031649
Solution 12	1271175	630607.5	625945	1244330	317103.8	629227.5	1251530	311262.5	1269975	630007.5	626365	311082.5	633967.5	1259535	313062.5	1245050	12570226	397752	0.031642
Solution 13	520475	255257.5	250595	493630	129428.8	253877.5	500830	123587.5	519275	254657.5	251015	123407.5	258617.5	508835	125387.5	494350	5063226	160457.2	0.031691
Solution 14	862275	426157.5	421495	835430	214878.8	424777.5	842630	209037.5	861075	425557.5	421915	208857.5	429517.5	850635	210837.5	836150	8481226	268478.2	0.031656
Solution 15	1145475	567757.5	563095	1118630	285678.8	566377.5	1125830	279837.5	1144275	567157.5	563515	279657.5	571117.5	1133835	281637.5	1119350	11313226	358009.5	0.031645
Solution 16	1482975	736507.5	731845	1456130	370053.8	735127.5	1463330	364212.5	1481775	735907.5	732265	364032.5	739867.5	1471335	366012.5	1456850	14688226	464719.6	0.031639

We analyzed Table 5 and Figures 3, 4, and 5 for finding the optimal solution, which are discussed as follows:

The mean objective function value of solution 16 that is obtained by scenario 16, is 14688226. This value is maximum profit mean among all solutions. So, we can select it as best-performed solution. It is illustrated in Figure 3.

Mean criteria is not enough for selecting optimal solution. Because in different conditions like uncertain situations there are fluctuations and we need a risk criteria to ensure reliability of decisions of decision makers (Ogryczak, 2000). We consider standard deviation (except variance) as risk criterion. Each of the solutions that have lower SD, it is more reliable in fluctuated environment. SD is achieved for all solutions of scenarios. Results are presented in Table 5 and Figure 4. According to the Table 5 and Figure 4, solution 1 has the minimum SD. So, it can be selected as more reliable in uncertain conditions. Regarding mean and SD criteria, there are two different optimal solutions. We can use coefficient of variation as an integrated approach to make final decision. Solution 16 has the minimum CV among all candidate solutions. It can be selected as the final optimal solution. It is presented in Figure 5.

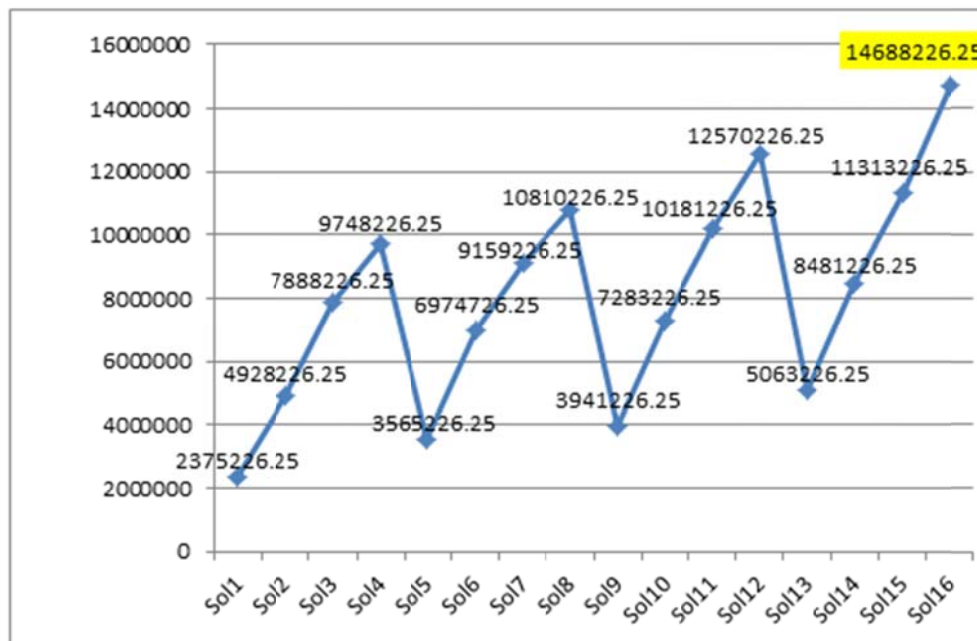


Figure 3: Mean results of objective values for candidate solutions

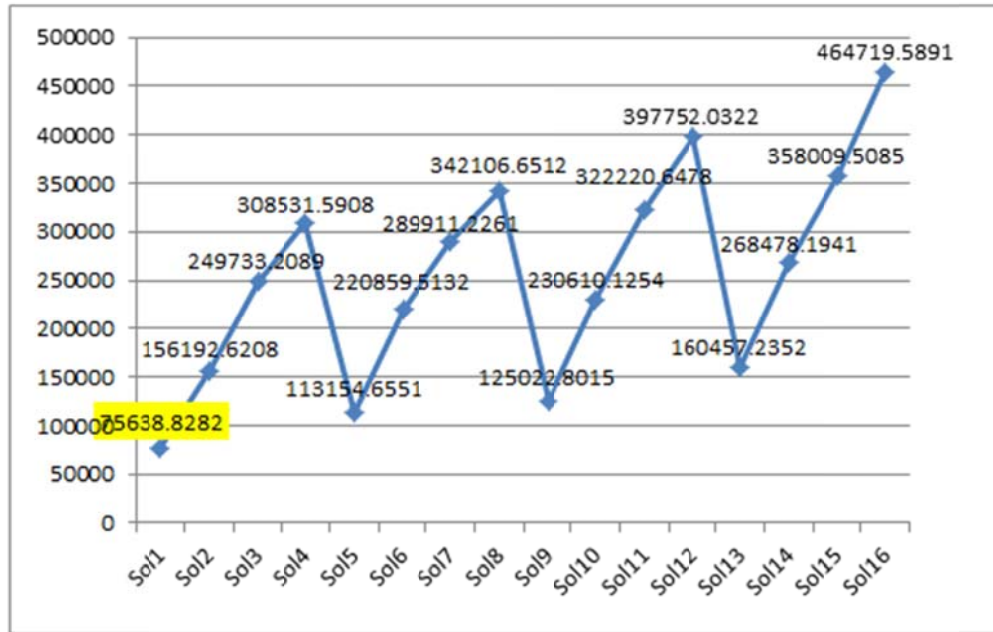


Figure 4: Standard deviation results of objective values for candidate solutions

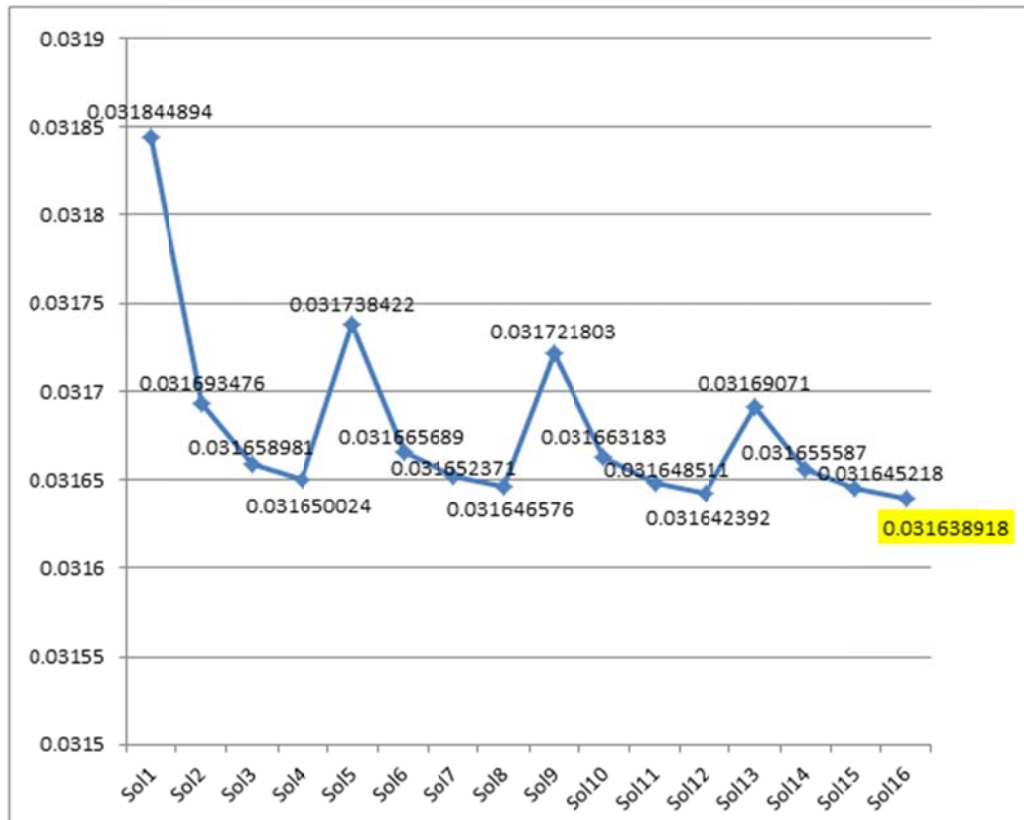


Figure 5: Coefficient of variation results of objective values for candidate solutions

## 5. Conclusion and future researches

In this paper, is considered a three-stage fixed charge transportation problem. The model is formulated as a mixed integer programming problem and is solved using a multi-criteria scenario

based solution approach to find optimal solution. First, 16 scenarios with different logical are generated randomly. Then, initial solutions of scenarios are evaluated and weighted average of the results is calculated. Finally, mean, standard deviation and coefficient of variation are regarded as acceptable criteria in order to decide about best solution. This model can be expanded to a multi-product and multi-period for the future research.

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